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REPRESENTATION OF DATA

TYPES OF VARIABLES

A variable is **qualitative** if it is not possible for it to take a numerical value.

A variable is **quantitative** if it can take a numerical value.

A quantitative variable which can take any value in a given range is **continuous**.

A quantitative variable which has clear steps between its possible values is **discrete.**

DISCRETE DATA

In a survey of 1 square meter pieces of land in a field, the number of snails in each of 30 pieces was recorded as follows:

1	2	4	0	2	3	1	4	2	3	5	2	2	3	2
2	3	1	2	3	2	0	1	1	2	0	3	2	3	3

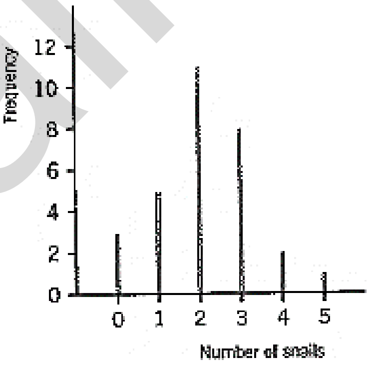
This is an example of discrete raw data

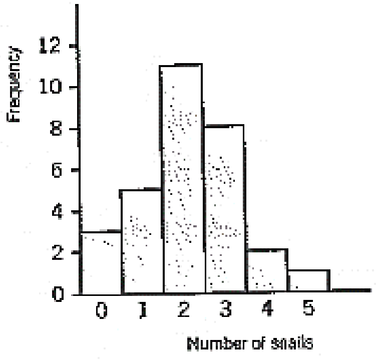
FREQUENCY DISTRIBUTION FOR DISCRETE DATA

To illustrate the data more concisely count the number of times each value occurs and summarize these in a table, known as a **frequency distribution**

Number of snails	0	1	2	3	4	5	
Frequency	3	5	11	8	2	1	Total 30

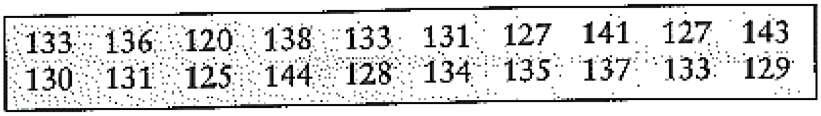
The frequency distribution can be represented diagrammatically by a vertical line graph or a bar chart. The height of the line or bar represents the frequency.





CONTINUOUS DATA

The following data were obtained in a survey of the heights of 20 children in a sports club. Each height was measured to the nearest centimeter.

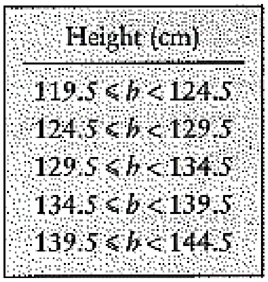


133	136	120	138	133	131	127	141	127	143
130	131	125	144	128	134	135	137	133	129

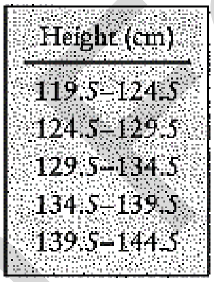
This is an example of **continuous raw data.**

FREQUENCY DISTRIBUTION FOR CONTINUOUS DATA

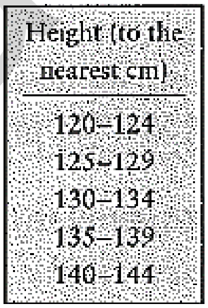
To form a frequency distribution of the heights of the 20 children, group the information into **classes** or intervals. Here are three different ways of writing the same set of intervals.



Height (cm)
$119.5 \leqslant h < 124.5$
$124.5 \leqslant h < 129.5$
$129.5 \leqslant h < 134.5$
$134.5 \leqslant h < 139.5$
$139.5 \leqslant h < 144.5$



Height (cm)
119.5–124.5
124.5–129.5
129.5–134.5
134.5–139.5
139.5–144.5



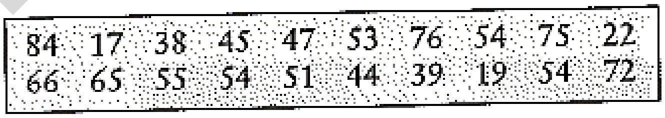
Height (to the nearest cm)
120–124
125–129
130–134
135–139
140–144

The values 119.5, 124.5, 129.5, … are called the **class boundaries** or the **interval boundaries**. The upper-class boundary (u.c.b.) of one interval is the lower-class boundary (l.c.b) of the next interval.

STEM AND LEAF DIAGRAMS

A very useful way of grouping data into classes while still retaining the original data is to draw a stem and leaf diagram, also known as a **stem plot.**

These are the marks of 20 students in an assignment:



84	17	38	45	47	53	76	54	75	22
66	65	55	54	51	44	39	19	54	72

Example 1.1

The tensile strength of 60 samples of rubber was measured and the results, in suitable units, were as follows.

174	160	141	153	161	159	163	186	179	167	154	145	156	159	171
156	142	169	160	171	188	151	162	164	172	181	152	178	151	177
180	186	168	169	171	168	157	166	181	171	183	176	155	161	182
160	182	173	189	181	175	165	177	184	161	170	167	180	137	143

Construct a stem-and-leaf diagram using two rows for each stem so that, for example, with a stem of 15 the first leaf may have digits 0 to 4 and the second leaf may have digits 5 to 9.

Example 1.2

Look at this stem and leaf diagram and for each of the three keys provided, give

a. The value ringed,
b. The width of the interval containing the ringed value

Stem	Leaf
0	7
0	9
1	0 1
1	2 2
1	4 4 4 5 5
1	6 6 7 7 7
1	8 8 8 8 9 9 ⑨
2	0 0 1 1
2	2 3
2	4

(i) the widths of 30 metal components

Key 1 | 2 means 1.2 cm

(ii) the reaction times of 30 volunteers

Key 1 | 2 means 12 hundredths of a second

(iii) the attendance at 30 matches

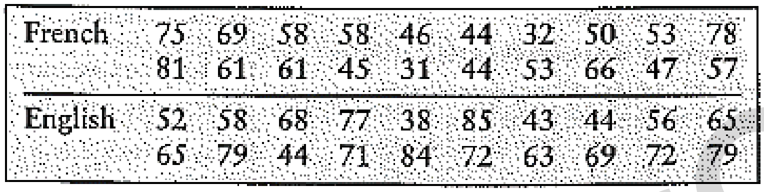
Key 1 | 2 means 1200 people

BACK-TO-BACK STEM PLOTS

Stem and leaf diagrams can be used to compare two samples by showing the results together on a back-to-back stem plot.

Example 1.3

Use a stem and leaf diagram to compare the examination marks in French and English for a class of 20 pupils

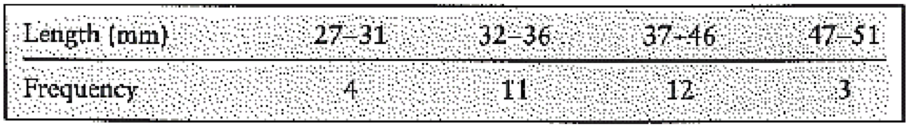


French	75	69	58	58	46	44	32	50	53	78
	81	61	61	45	31	44	53	66	47	57
English	52	58	68	77	38	85	43	44	56	65
	65	79	44	71	84	72	63	69	72	79

WAYS OF GROUPING CONTINUOUS DATA

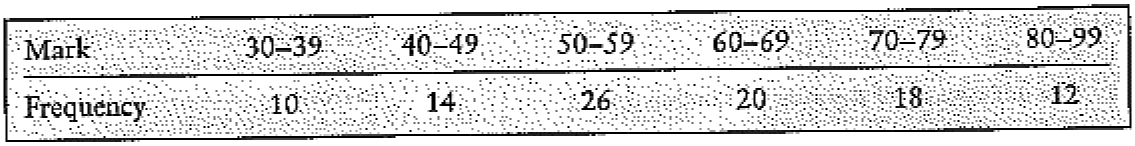
The following frequency distributions show some of the ways that data can be grouped. The information is more concise than the raw data, but the disadvantage is that the original information has been lost.

(i) Frequency distribution to show the lengths, to the nearest millimeter, of 30 rods



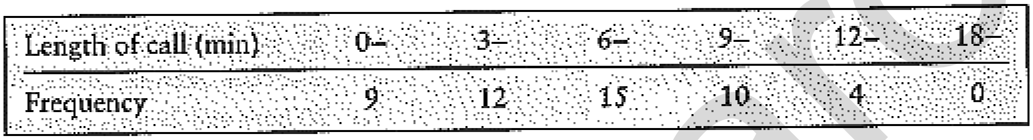
Length (mm)	27–31	32–36	37–46	47–51
Frequency	4	11	12	3

(ii) Frequency distributions to show the lengths the marks in a test of 10 students



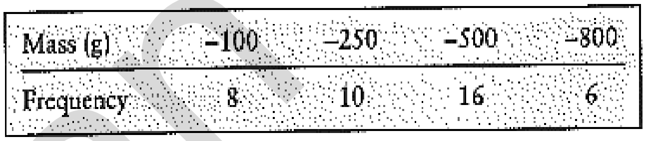
Mark	30–39	40–49	50–59	60–69	70–79	80–99
Frequency	10	14	26	20	18	12

(iii) Frequency distribution to show the lengths the marks in a test of 10 students



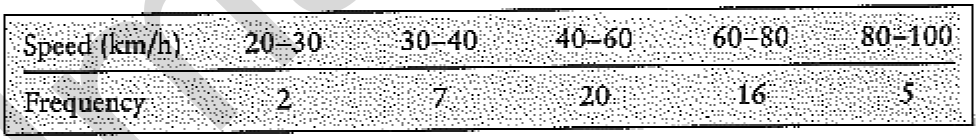
Length of call (min)	0–	3–	6–	9–	12–	18–
Frequency	9	12	15	10	4	0

(iv) Frequency distribution to show the masses of 40 packages brought to a particular counter at a post office



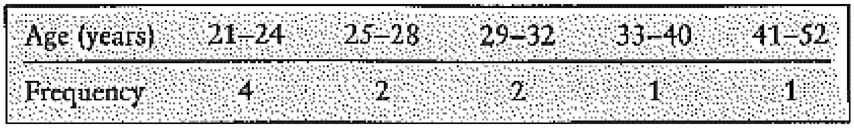
Mass (g)	–100	–250	–500	–800
Frequency	8	10	16	6

(v) Frequency distribution to show the speeds of 50 cars passing a checkpoint



Speed (km/h)	20–30	30–40	40–60	60–80	80–100
Frequency	2	7	20	16	5

(vi) Frequency distribution to show ages (in completed years) of applicants for a teaching post



Age (years)	21–24	25–28	29–32	33–40	41–52
Frequency	4	2	2	1	1

WIDTH OF AN INTERVAL

The width of an interval is the difference between the boundaries

Width of an interval = upper class boundary – lower class boundary

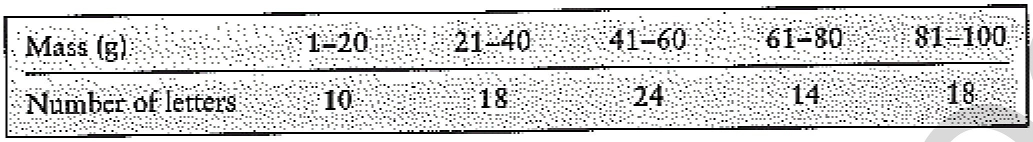
HISTOGRAMS

Grouped data can be displayed in a histogram as in the following diagram

Age, x years	$0 \leqslant x < 20$	$20 \leqslant x < 40$	$40 \leqslant x < 50$	$50 \leqslant x < 70$	$70 \leqslant x < 100$
Frequency	4	44	36	28	6

Example 1.4

The grouped frequency distribution records the masses, to the nearest gram, of 84 letters delivered by the postman.

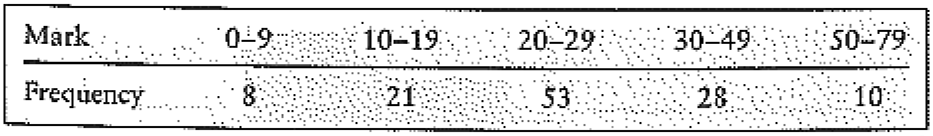


Mass (g)	1–20	21–40	41–60	61–80	81–100
Number of letters	10	18	24	14	18

Draw a histogram to illustrate these data.

Example 1.5

These are the examination marks for a group of 120 first year statistics students.



Mark	0–9	10–19	20–29	30–49	50–79
Frequency	8	21	53	28	10

FINDING THE FREQUENCIES FROM A HISTOGRAM

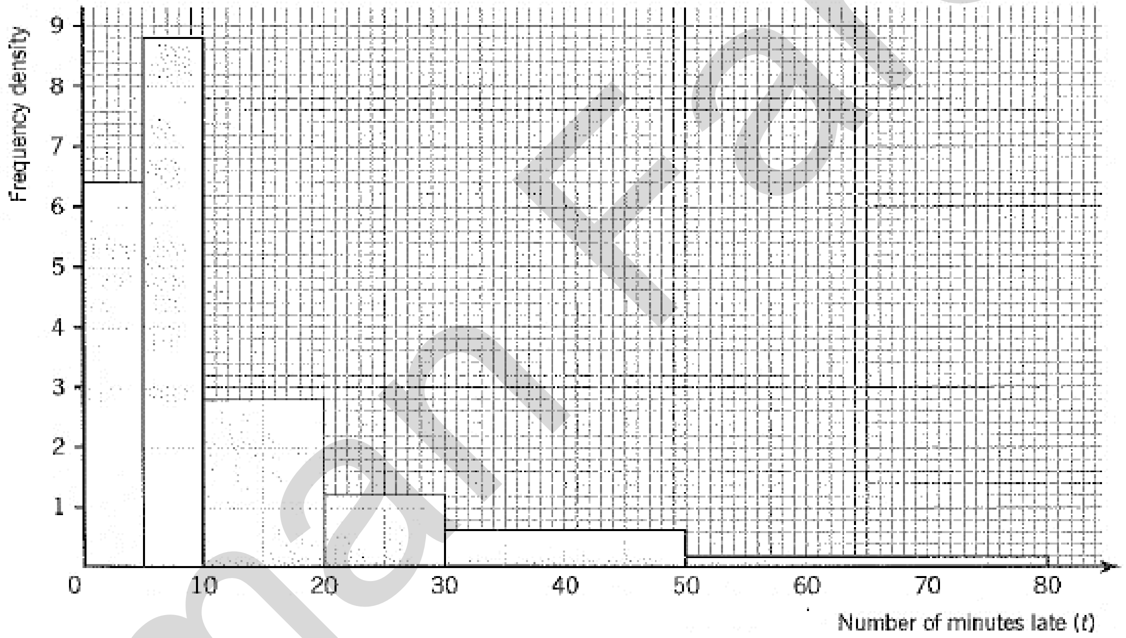
To find the frequency in each interval, use

Frequency = interval width x frequency density

Example 1.6
A Passengers' Association conducted a survey on the punctuality of trains using a particular station. The histogram illustrates the results.

(a) Construct the frequency distribution.
(b) How many trains were there in the survey?

Histogram to show lateness of trains



FREQUENCY POLYGONS

A grouped frequency distribution can be displayed as a frequency polygon.

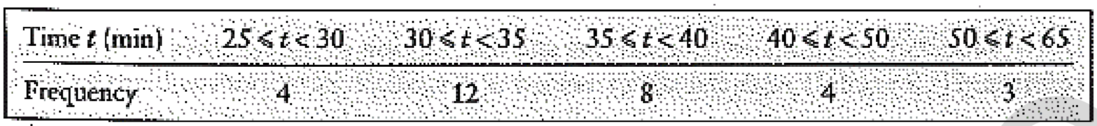
To construct a frequency polygon, for each interval plot frequency density against the mid-interval value, where

$$\text{Mid interval value} = \frac{1}{2}(\text{lower class boundary} + \text{upper class boundary})$$

Then join the points with straight lines.

Example 1.7

Draw a frequency polygon to illustrate this frequency distribution which gives the times taken by 31 competitors to complete a cross-country run.



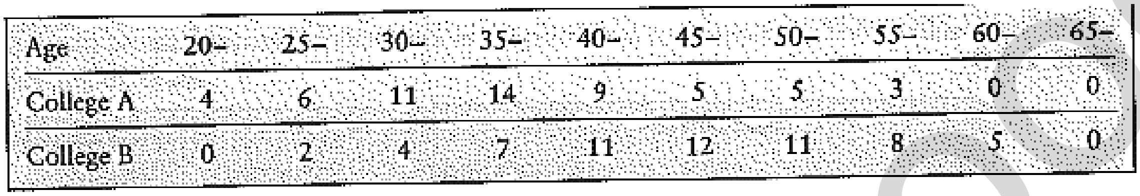
Time t (min)	$25 \leqslant t < 30$	$30 \leqslant t < 35$	$35 \leqslant t < 40$	$40 \leqslant t < 50$	$50 \leqslant t < 65$
Frequency	4	12	8	4	3

COMPARATIVE FREQUENCY POLYGONS

Frequency polygons are very useful when comparing sets of data.

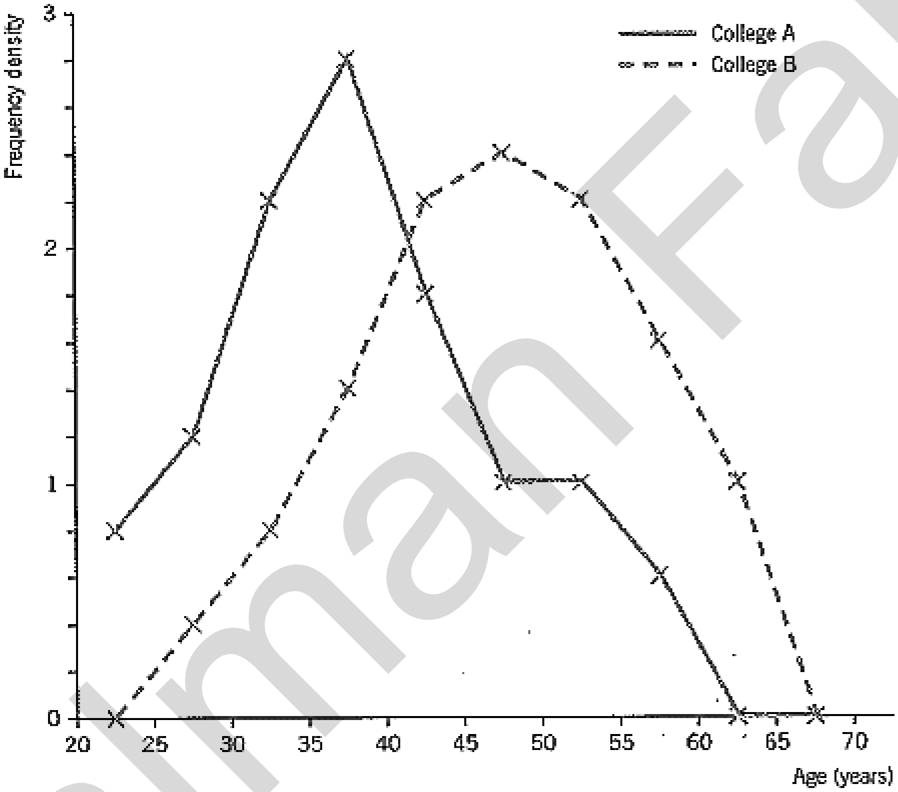
Example 1.8

Draw frequency polygons to compare the age distribution of the teachers in two sixth form colleges:



Age	20–	25–	30–	35–	40–	45–	50–	55–	60–	65–
College A	4	6	11	14	9	5	5	3	0	0
College B	0	2	4	7	11	12	11	8	5	0

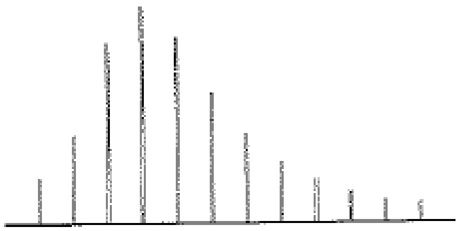
Solution:

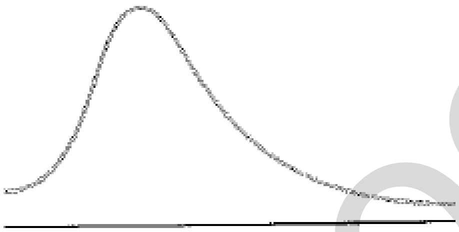


THE SHAPE OF DISTRIBUTIONS

If distributions represented by a vertical line graph or a histogram are illustrated using a frequency curve, it is easier to see the general 'shape' of the distribution.

a) Positive skew.

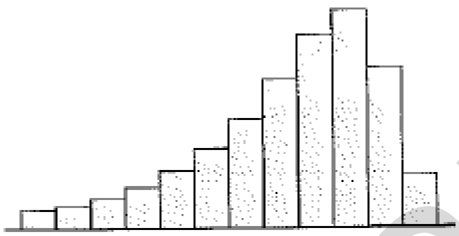


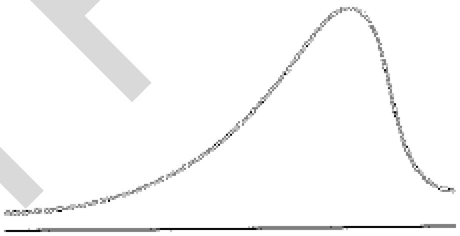


In a positive skewed distribution, there is a long tail at the positive end of the distribution.
A positively skewed distribution could occur when considering, for example,

- The number of children in a family,
- The age at which women marry
- the distribution of wages in a firm.

b) Negative skew.

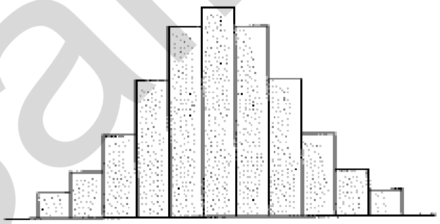




In a negatively skewed distribution, there is a long tail at the negative end of the distribution.
A negatively skewed distribution could occur when considering, for example,

- reaction times for an experiment
- daily maximum temperatures

c)The normal distribution



This symmetrical, bell-shaped distribution is known as a normal distribution. An approximately normal distribution occurs when measuring quantities such as heights, masses, examination marks.

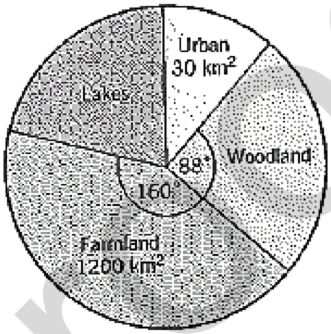
CIRCULAR DIAGRAMS OF PIE CHARTS

Pie charts are so called because they look like an apple pie! The areas of the slices or sectors of the pie are in proportion to the quantities being represented.

Example 1.9

The pie chart, which is not drawn to scale, shows the distribution of various types of land and water in a certain county. Calculate

(a) The area of woodland,
(b) The angle of the urban sector,
(c) The total area of the county.



INTERPRETATION OF DATA

THE MODAL CLASS

The highest bar in the histogram represents the interval 40 < x < 50, This is the modal class. Notice that in the table this interval does not have the greatest frequency, but it does have the greatest frequency density.

In a grouped frequency distribution, the modal class is the interval with the greatest frequency density, i.e. the interval represented by the highest bar in the histogram.

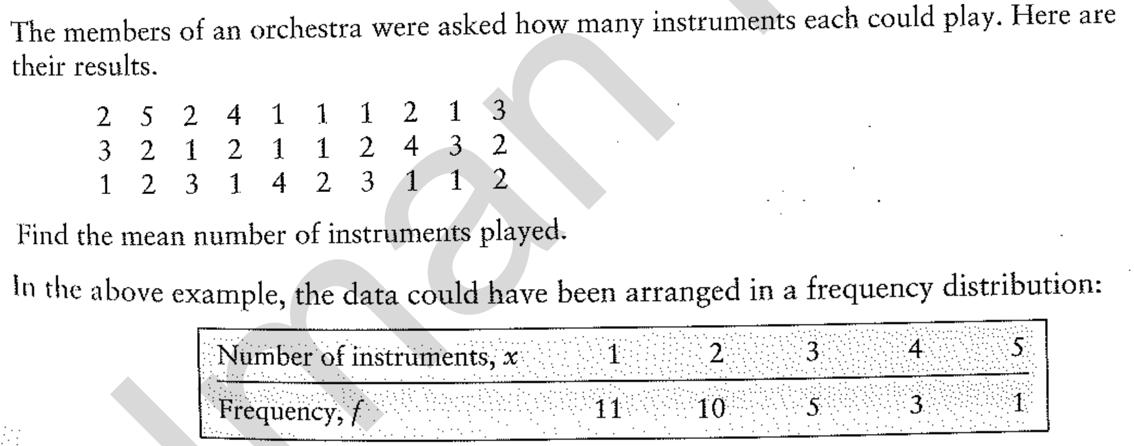
THE MEAN

A typical or average value is useful when interpreting data. One such average is the mean.
Consider the five number

0.9, 1.4, 2.8, 3.1, 5.6.

Example 1.10

To obtain Grade A, Ben must achieve an average of at least 70 in five tests. If his average mark for the first four test is 68, what is the lowest mark he can get in his fifth test and still obtain Grade A?



The members of an orchestra were asked how many instruments each could play. Here are their results.

2	5	2	4	1	1	1	2	1	3
3	2	1	2	1	1	2	4	3	2
1	2	3	1	4	2	3	1	1	2

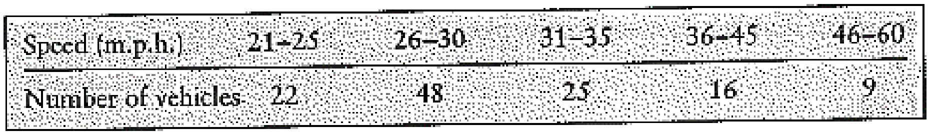
Find the mean number of instruments played.

In the above example, the data could have been arranged in a frequency distribution:

Number of instruments, x	1	2	3	4	5
Frequency, f	11	10	5	3	1

Example 1.11

The speeds, to the nearest mile per hour, of 120 vehicles passing a check point were recorded and are grouped in the table below.



Speed (m.p.h.)	21–25	26–30	31–35	36–45	46–60
Number of vehicles	22	48	25	16	9

Estimate the mean of this distribution.

VARIABILITY OF DATEA

Each of these sets of number has a mean of 7 but the spread of each is set is different

(a) 7,7,7,7,7
(b) 4,6, 6.5, 7.2, 11.3
(c) $-193, -46,\ 28,\ 69, 177$

The range

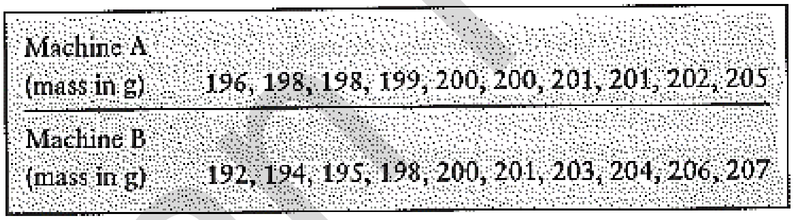
The range is based entirely on the extreme values of the distribution.

Range = highest value – lowest value

THE STANDARD DEVIATION, s, AND THE VARIANCE, s^2

Examples 1.12

Two machines, A and B, are used to pack biscuits. A random sample of ten packets was taken from each machine and the mass of each packet was measured to the nearest gram and noted. Find the standard deviation of the masses of the packets taken in the sample from each machine, Comment on your answer.



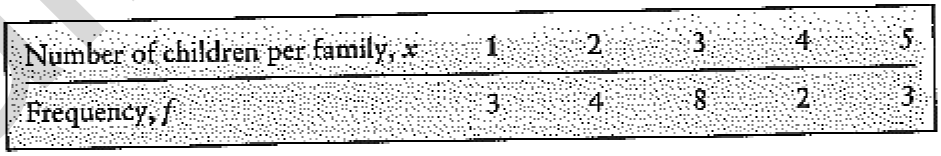
Machine A (mass in g)	196, 198, 198, 199, 200, 200, 201, 201, 202, 205
Machine B (mass in g)	192, 194, 195, 198, 200, 201, 203, 204, 206, 207

Example 1.13

The mean of the five numbers 2,3,5,6,8 is 4.8. Calculate the standard deviation.

Example 1.24

Consider again the data ,which shows the number of children in 20 families. The mean is 2.9.



Number of children per family, x	1	2	3	4	5
Frequency, f	3	4	8	2	3

Example 1.14

(a) Cartons of orange juice are advertised as containing 1 litre. A random sample of 100 cartons gave the following results for the volume, x.

$\sum x = 101.4, \sum x^2 = 102.83$

Calculate the mean and the standard deviation of the volume of orange juice in these 100 cartons.

(b) A machine is supposed to cut lengths of rod 50 cm long,

A sample of 20 rods gave the following results for the length, x.

$\sum fx = 997, \sum fx^2 = 49711$

(i) Calculate the mean length of the 20 rods.

(ii) Calculate the variance of the lengths of the 20 rods.

CALCULATIONS INVOLVING THE MEAN AND THE STANDARD DEVIATION

Example 1.15

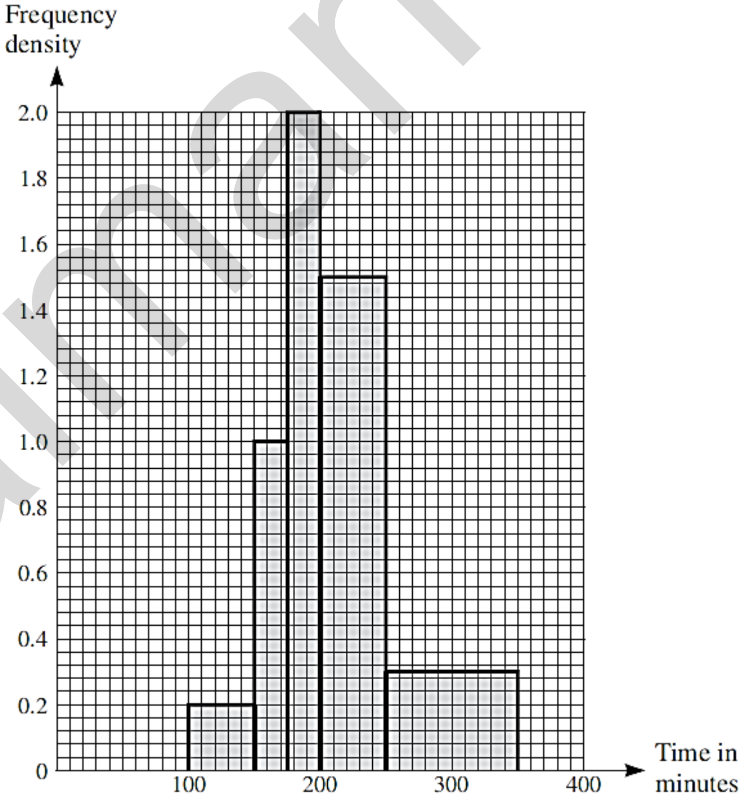
(a) Calculate the mean and the standard deviation of the four numbers 2,3,6,9.

(b) Two numbers, a and b, are to be added to this set of four numbers, such that the mean is increased by 1 and the variance is increased by 2.5. Find a and b.

Example 1.16

N13/62/Q4

The following histogram summarises the times, in minutes, taken by 190 people to complete a race.



(i) Show that 75 people took between 200 and 250 minutes to complete the race. [1]

(ii) Calculate estimates of the mean and standard deviation of the times of the 190 people. [6]

(iii) Explain why your answers to part **(ii)** are estimates. [1]

Example 1.17

J14/62/Q6

The times taken by 57 athletes to run 100 metres are summarised in the following cumulative frequency table.

Time (seconds)	<10.0	<10.5	<11.0	<12.0	<12.5	<13.5
Cumulative frequency	0	4	10	40	49	57

(i) State how many athletes ran 100 metres in a time between 10.5 and 11.0 seconds. [1]

(ii) Draw a histogram on graph paper to represent the times taken by these athletes to run 100 metres. [4]

(iii) Calculate estimates of the mean and variance of the times taken by these athletes. [4]

COMBINING SETS OF DATA

Example 1.18

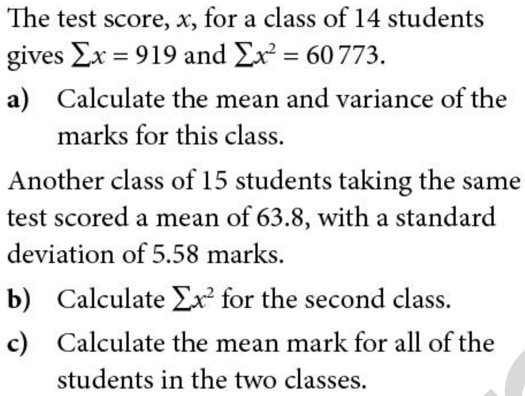
The number of errors, x, on each of 200 pages of typescript was monitored. The results when summarized showed that

$\sum x = 920 \quad \sum x^2 = 5032$

(a) Calculate the mean and the standard deviation of the number of errors per page. A further 50 pages were monitored it was found that the mean was 4.4 errors and the standard deviation was 2.2 errors.

(b) Find the mean and the standard deviation of the number of errors per page for the 250 pages.

Example 1.19



The test score, x, for a class of 14 students gives $\sum x = 919$ and $\sum x^2 = 60\,773$.

a) Calculate the mean and variance of the marks for this class.

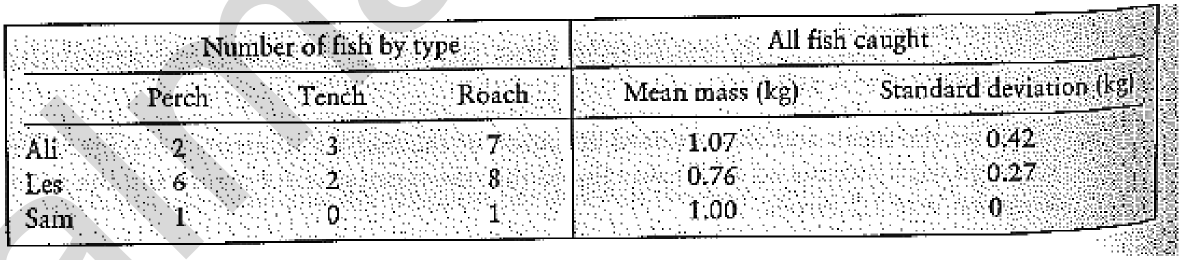
Another class of 15 students taking the same test scored a mean of 63.8, with a standard deviation of 5.58 marks.

b) Calculate $\sum x^2$ for the second class.

c) Calculate the mean mark for all of the students in the two classes.

Example 1.20

Three statistics students, Ali, Les and Sam, spent the day fishing. They caught three different types of fish and recorded the type and mass (correct to the nearest 0.01kg) of each fish caught, At 4pm, they summarized the results as follows.



	Number of fish by type			All fish caught	
	Perch	Tench	Roach	Mean mass (kg)	Standard deviation (kg)
Ali	2	3	7	1.07	0.42
Les	6	2	8	0.76	0.27
Sam	1	0	1	1.00	0

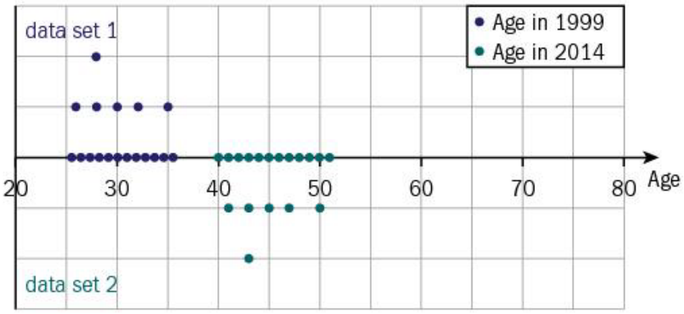
(a) State how it may be deduced from the data that the mass of each fish caught by Sam was 1.00kg.

(b) The winner was the person who had caught the greatest total mass of fish by 4pm. Determine who was the winner, showing your working.

(c) Before leaving the waterside, Sam catches one more fish and weighs it. He then announces that, if this extra fish is included with the other two fish he caught, the standard deviation is 1.00kg. Find the mass of this extra fish.

USING A METHOD OF CODING TO FIND THE MEAN AND STANDARD DEVIATION

The diagram shows two sets of data:
Data set 1 represents the ages, in years, of a group of people; Data set 2 represents the ages of the same people 15 years later.



All the data values have shifted by 15, so

Mean of Data set 2 = (mean of Data set 1) + 15

and there is no change to any measure of spread.

Example 1.21

Salt is packed in bags which the manufacturer claims contains 25kg each. Eighty bags are examined and the mass, x kg, of each is found. The results are $\sum(x-25)=27.2$, $\sum(x-25)^2=85.1$. Find the mean and the standard deviation of the masses.

Example 1.22

The weights (in kg) of the hand luggage carried on to a flight by 97 passengers are summarised by

$\sum(x-5)=314 \quad \sum(x-5)^2=1623.$

Find $\sum x$ and $\sum x^2$.

Example 1.23

A summary of 20 observations of x gave the following information:

$\sum(x-a)=-23.2$ and $\sum(x-a)^2=211.23$

The mean of these values of x is 8.95.

i) Find the value of the constant a.

ii) Find the standard deviation of these values of x.

Example 1.24 - 61/J13/2

A summary of 30 values of x gave the following information:

$$\Sigma(x-c) = 234, \qquad \Sigma(x-c)^2 = 1957.5,$$

where c is a constant.

(i) Find the standard deviation of these values of x. [2]

(ii) Given that the mean of these values is 86, find the value of c. [2]

Example 1.25 - 63/N14/2

A traffic camera measured the speeds, x kilometres per hour, of 8 cars travelling along a certain street, with the following results.

62.7 59.6 64.2 61.5 68.3 66.9 62.0 62.3

(i) Find $\Sigma(x-62)$. [1]

(ii) Find $\Sigma(x-62)^2$. [1]

(iii) Find the mean and variance of the speeds of the 8 cars. [3]

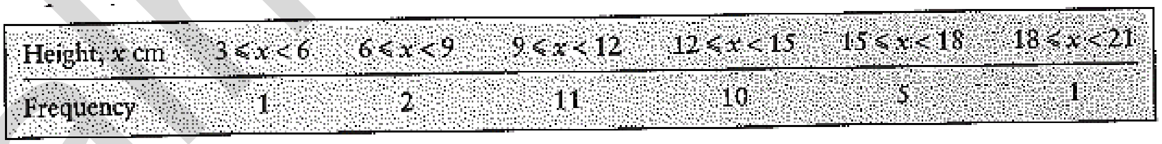
CUMULATIVE FREQUENCY

The cumulative frequency is the total frequency up to a particular item. A cumulative frequency distribution can be obtained from a frequency distribution and can be illustrated,

Cumulative frequency curves for grouped data

Consider this situation:

Six weeks after planting, the heights of 30 broad bean plants were measured and the frequency distribution formed as shown.



Height, x cm	$3 \leqslant x < 6$	$6 \leqslant x < 9$	$9 \leqslant x < 12$	$12 \leqslant x < 15$	$15 \leqslant x < 18$	$18 \leqslant x < 21$
Frequency	1	2	11	10	5	1

The cumulative frequency is calculated up to each upper class boundary

This can be shown diagrammatically in a cumulative frequency graph.

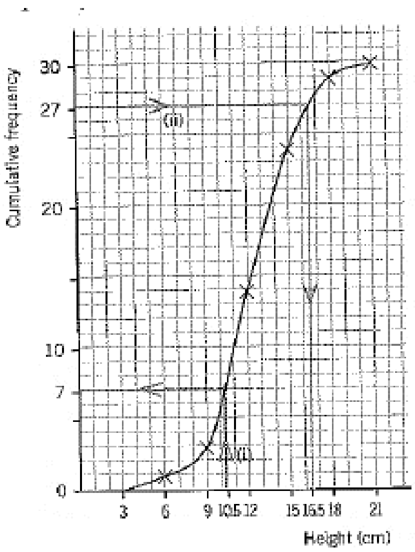
The cumulative frequency is plotted against the upper class boundaries and the points are joined as follows:

(i) for a cumulative frequency polygon,

- join the points with straight lines, indicating that you are assuming that the readings are evenly distributed throughout the interval. This ties in with the fact that you draw horizontal lines at the tops of the blocks in a histogram.

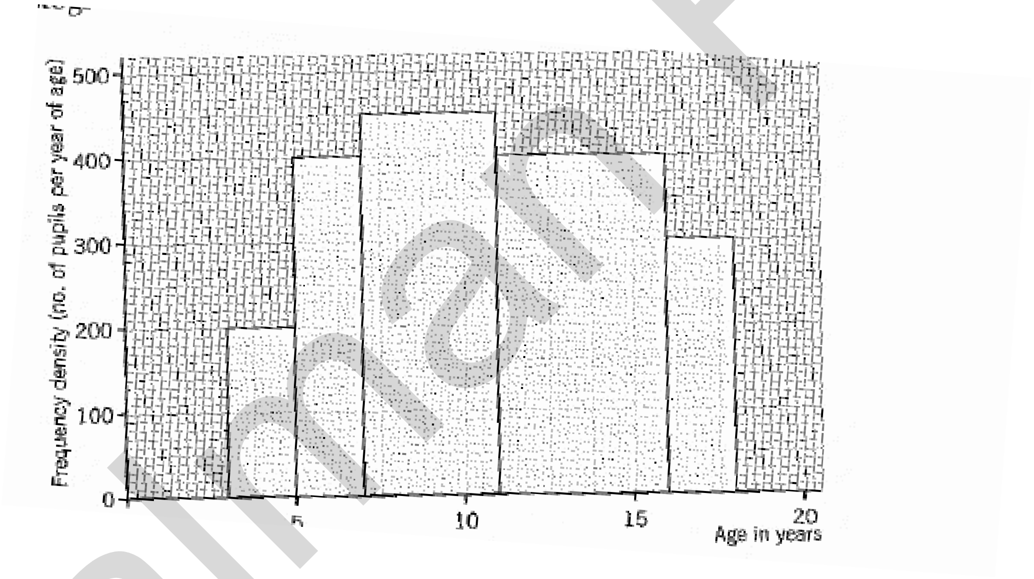
(ii) for a cumulative frequency curve,

- join the points with a smooth curve, in this case you are assuming a distribution of readings throughout the interval which not be even.



Example 1.26

A survey is carried out to determine the numbers of pupils in various age group who are attending nurseries, schools and colleges within a certain area. The results are summaries in the following histogram.



a) Copy and complete the following table showing the ages of the pupils and the corresponding cumulative frequencies.

Age in years up to	3	5	7	11	16	18
Cumulative frequency	0					5600

b) Draw a cumulative frequency diagram for the distribution.

c) Use your cumulative frequency diagram to estimate the age exceeded by 30% of the pupils in the survey.

Example 1.27

Students were asked how long it took them to travel to college on a particular morning. A cumulative frequency distribution was formed:

Time taken (minutes)	Cumulative frequency
<5	28
<10	45
<15	81
<20	143
<25	280
<30	349
<35	374
<40	395
<45	400

a) Draw a cumulative frequency polygon.
b) Estimate how many students took less than 18 minutes.
c) Taking equal class intervals of 0- ,5- 10-, …., construct a frequency distribution and draw a histogram.

MEDIANS, QUARTILES AND PERCENTAGES

Ungrouped data – median

For ungrouped data consisting of n observations in order of size, the median is the $\frac{1}{2}(\boldsymbol{n}+\mathbf{1})^{\boldsymbol{th}}$ observation.

(a) Consider this set of numbers 7, 7, 2, 3, 4, 2, 7, 9, 31.

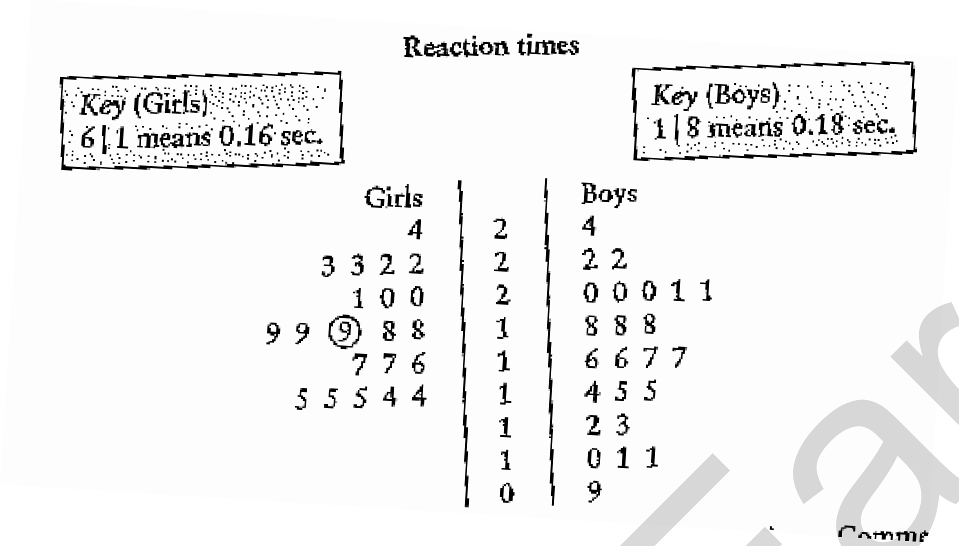
(b) Consider this set of numbers: 36, 41, 27, 32, 29, 39, 39, 43.

Note that

- If there is an odd number of observations, the median is the middle value,
- If there is an even number of observations, there are two middle values. If these are c and d, the median is halfway between them, i.e. $\frac{1}{2}(c+d)$

Example 1.28

A reaction time experiment was performed first with 21 girls and then with 24 boys, The results are shown on the stem and leaf diagram.



Reaction times

Key (Girls)
6 | 1 means 0.16 sec.

Key (Boys)
1 | 8 means 0.18 sec.

Girls		Boys
4	2	4
3 3 2 2	2	2 2
1 0 0	2	0 0 0 1 1
9 9 ⑨ 8 8	1	8 8 8
7 7 6	1	6 6 7 7
5 5 5 4 4	1	4 5 5
	1	2 3
	1	0 1 1
	0	9

Find the median and the interquartile range for both sets of reaction times. Comment on your answers.

Example 1.29

N12/61/Q4

Prices in dollars of 11 caravans in a showroom are as follows.

16 800 18 500 17 700 14 300 15 500 15 300 16 100 16 800 17 300 15 400 16 400

(i) Represent these prices by a stem-and-leaf diagram. [3]

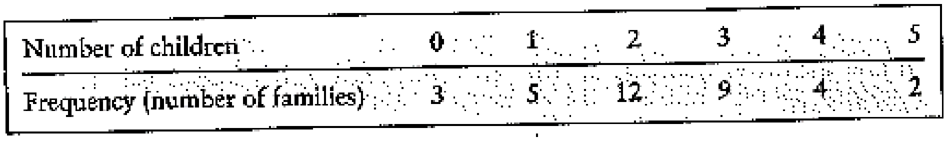
(ii) Write down the lower quartile of the prices of the caravans in the showroom. [1]

Ungrouped data in a frequency distribution – median and quartiles

To find the median and the quartiles of data in the form of an ungrouped frequency distribution, it is useful to find the cumulative frequency as thus gives the total frequency up to a particular item.

Example 1.30

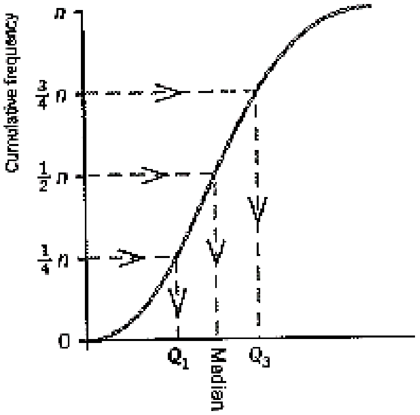
The table shows the number of children in the family for 35 families in a certain area. Find the median number of children per family, and the interquartile range.

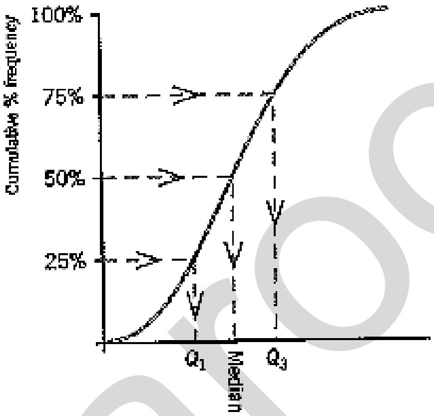


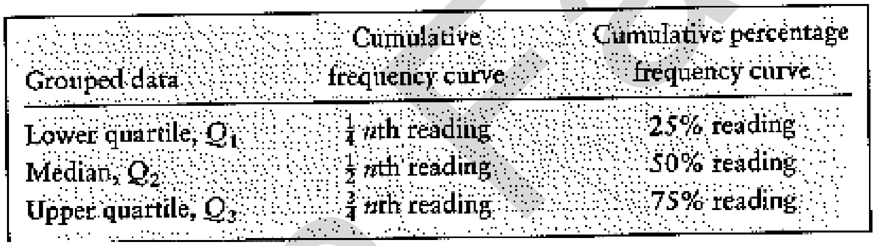
Number of children	0	1	2	3	4	5
Frequency (number of families)	3	5	12	9	4	2

Grouped Data – Median and Quartiles

When data have been grouped into intervals, the original information has been lost, so it is only possible to make estimates of the median and quartiles. One way of doing this is to use a cumulative frequency graph, or cumulative percentage frequency graph as follows:



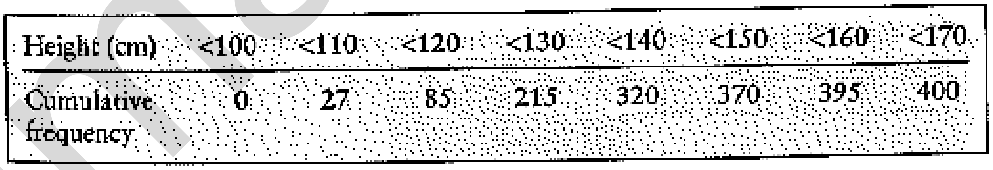




Grouped data	Cumulative frequency curve	Cumulative percentage frequency curve
Lower quartile, Q_1	$\frac{1}{4}$ *n*th reading	25% reading
Median, Q_2	$\frac{1}{2}$ *n*th reading	50% reading
Upper quartile, Q_3	$\frac{3}{4}$ *n*th reading	75% reading

Example 1.31

The table gives the cumulative distribution of the heights (in centimeters) of 400 children in a certain school:



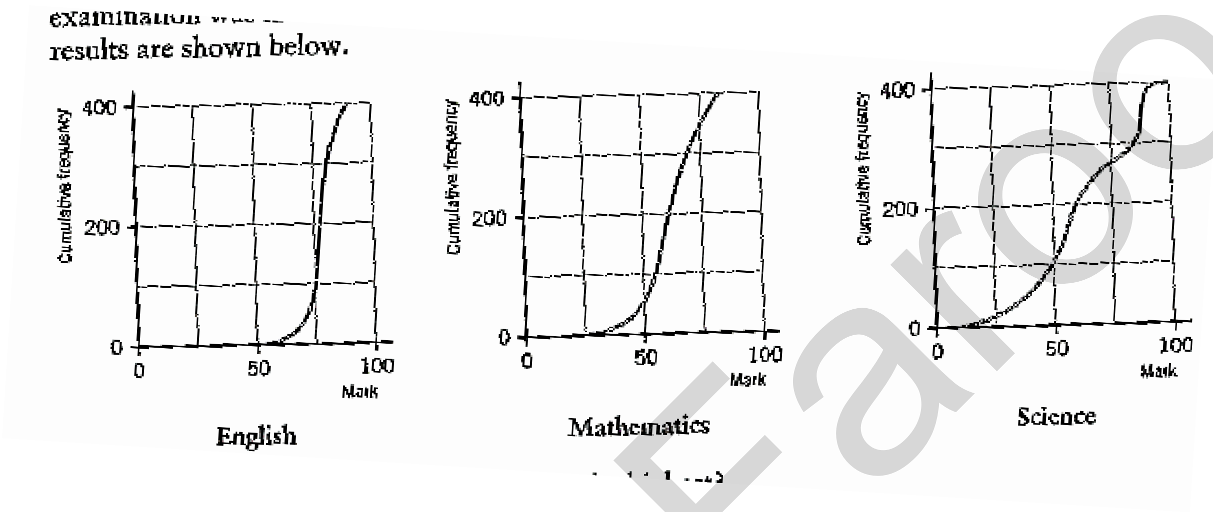
Height (cm)	<100	<110	<120	<130	<140	<150	<160	<170
Cumulative frequency	0	27	85	215	320	370	395	400

a) Draw a cumulative frequency curve.
b) Find an estimate of the median height,
c) Determine the interquartile range.

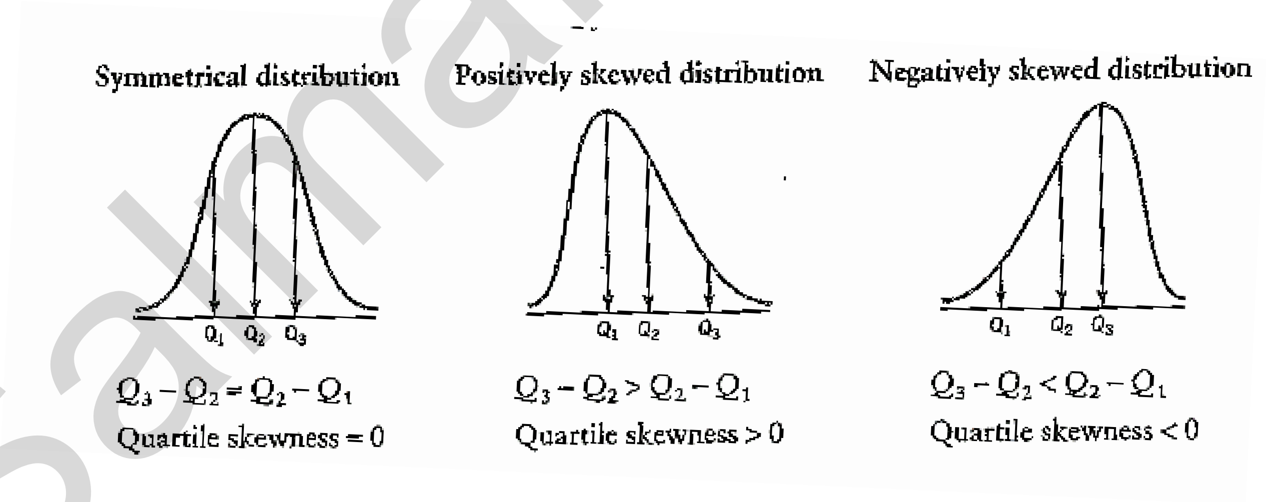
SKEWNESS

Example 1.32

Examinations in English, Mathematics and Science were taken by 400 students. Each examination was marked out of 100 and the cumulative frequency graphs illustrating the results are shown below.



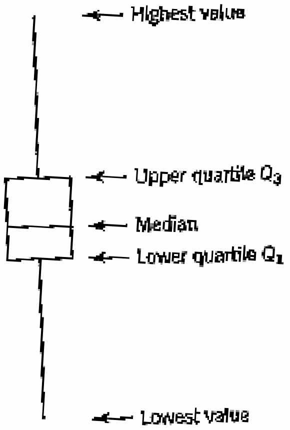
(a) In which subject was the median mark the highest?
(b) In which subject was the interquartile range of the marks the greatest?
(c) In which subject did approximately 75% of the students score 50 marks or more?



BOX AND WHISKERS PLOT

The box and whisker diagram, or box plot, illustrates the dispersion, or spread of the distribution. It uses the highest and lowest values of data, the quartiles (Q_1 and Q_3) and the median (Q_2). For example:

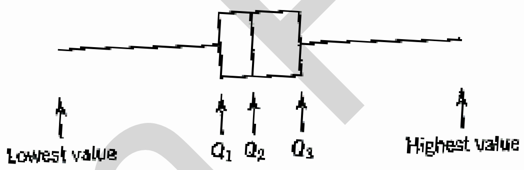
Vertically



Notice that the 'box' extends from Q_1 to Q_3 and so encloses the middle 50% of the data.

The 'whiskers' extend from the box to the highest and lowest values and illustrate the range of the data.

Horizontally



A box plot for a symmetrical distribution would look like this:

Example 1.33

A class of pupils played a computer game which tested how quickly they reacted to a visual instruction to press a particular key. The computer measured their reaction times in tenths of a second and stored a record of the gender and reaction time of each pupil. Finally it displayed the following summary statistics for the whole class.

	Median	Lower quartile	Upper quartile	Min	Max
Girls	10	8	15	6	19
Boys	10	7	13	4	16

(a) Draw two box plots suitable for comparing the reaction times of boys and girls.
(b) Write a brief comparison of the performance of boys and girls in this game.

Example 1.34

A group of children carry out a survey of the numbers of sweets in each of 50 packets. Their results are shown in the following stem and leaf diagram.

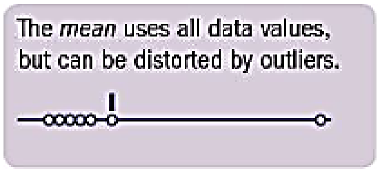
20	4 6 6 7 7 7 7 8 8 9 9 9 9
30	0 0 0 0 0 1 1 2 2 2 3 3 4 4 4
30	5 5 5 6 6 7 7 7 8 8 8 8 8 9 9
40	0 0 1 1 2 4 4

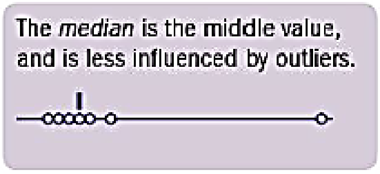
Key: 20 | 4 means 24 sweets in a packet

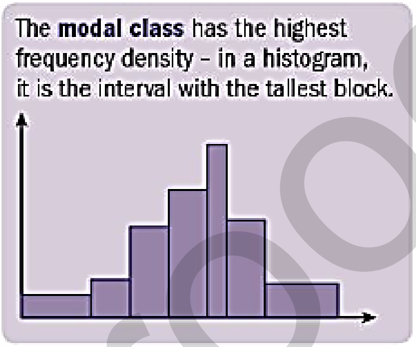
(a) Calculate the median and the quartiles of this distribution.
(b) Draw a box plot for the distribution.

COMPARING DISTRIUTIONS

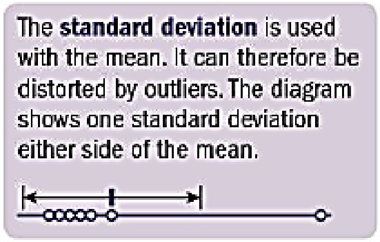
There are various measures of average that you can use to compare distributions, and you are not expected to calculate all of them.

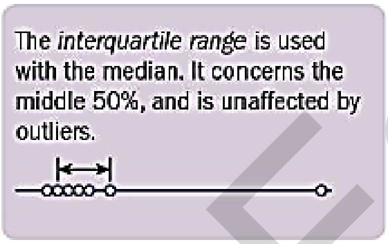

The *mean* uses all data values, but can be distorted by outliers.

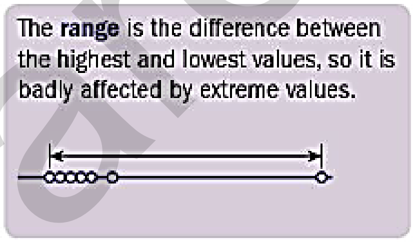

The *median* is the middle value, and is less influenced by outliers.


The **modal class** has the highest frequency density – in a histogram, it is the interval with the tallest block.

Similarly there are various measures of spread as well.


The **standard deviation** is used with the mean. It can therefore be distorted by outliers. The diagram shows one standard deviation either side of the mean.

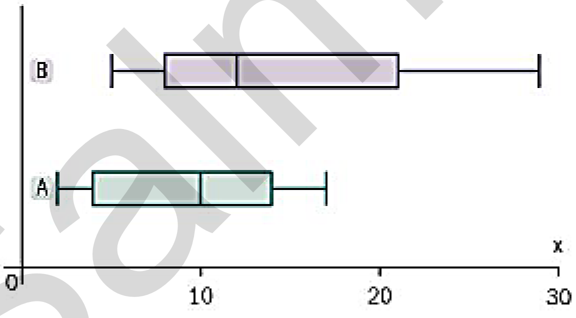

The *interquartile range* is used with the median. It concerns the middle 50%, and is unaffected by outliers.


The **range** is the difference between the highest and lowest values, so it is badly affected by extreme values.

- If you are comparing distributions,
  - make the comparison in context
  - make reference to both the average values and the spread.

It is easier to make comparisons between two distributions than it is to describe one set of data from a graph. Histograms, box-and-whisker and stem-and-leaf diagrams all show average values, spread and whether the distribution is reasonably symmetric.

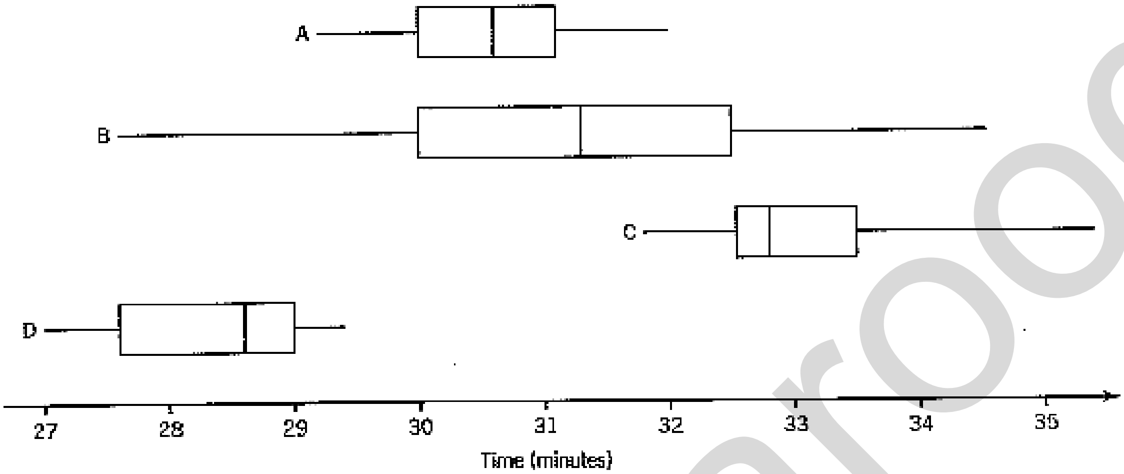
The following data are marks on a test out of 30 for two classes, A and B.



On average, class B have done better than class A. There is a much greater spread of marks in class B than in class A.
The other striking feature here is that the top half of the marks in class B are very spread out in comparison with the bottom half of B and both halves of A.

Example 1.35

A group of athletes frequently run round a cross-country course in training. The box and whisker plots below represent the times taken by athletes A, B, C and D to complete the course.



(a) Compare the times taken by athletes C and D.

Assume that the distributions shown above are representative of the times the athletes would take in a take in a race over the same course.

(b) Which of the athletes A or B would you choose of you were asked to select one of them to win a race against

(i) C

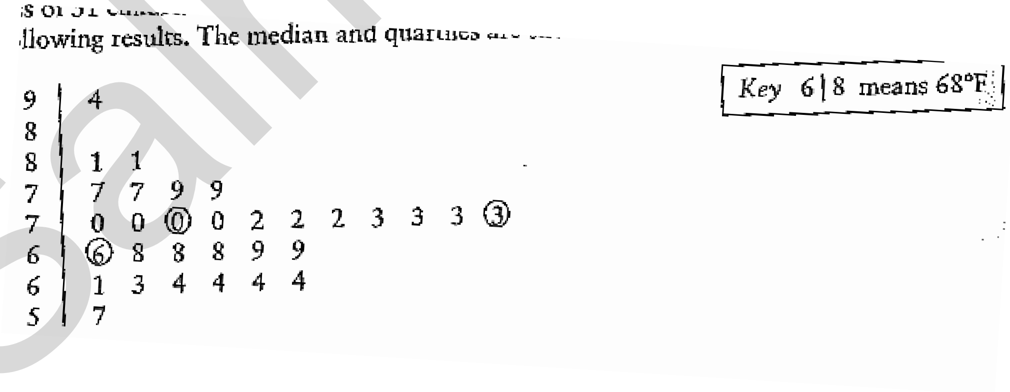
(ii) D?

Give a reason for each answer.

(c) Which athlete would be most likely to win a race between A and B?

Example 1.36

A class of 31 recorded the maximum daily temperature for the month of July with the following results. The median and quartiles are shown on the stem and leaf diagram,



s of 31

llowing results. The median and quartiles

Key 6|8 means 68°F

9	4
8	
8	1 1
7	7 7 9 9
7	0 0 ⓪ 0 2 2 2 3 3 3 ③
6	⑥ 8 8 8 9 9
6	1 3 4 4 4 4
5	7

(a) by using the values of the quartiles and illustrating your results on a boxplot,

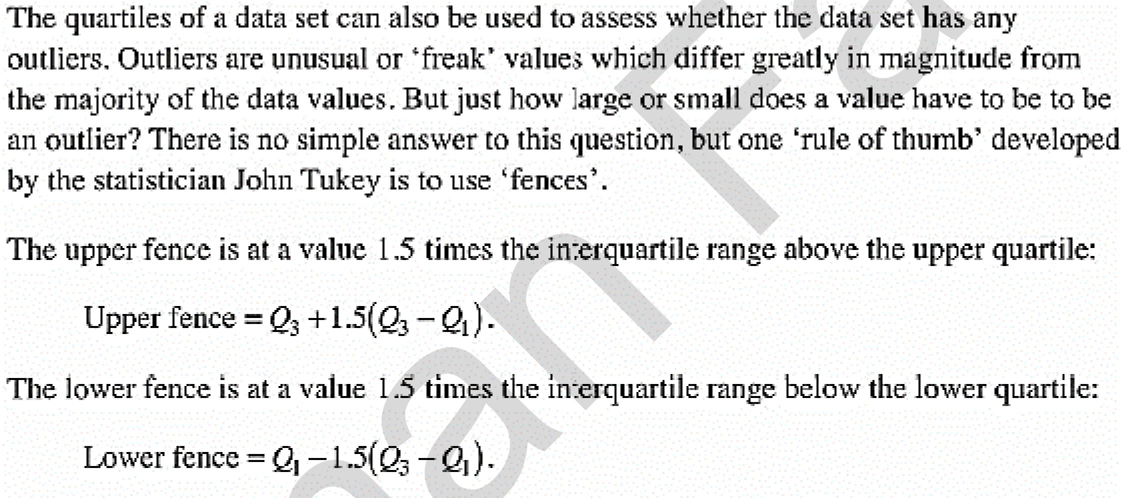
(b) by using the mean and the standard deviation.

Example 1.37

The time t (in seconds) taken by an athlete to run 400 meters on ten successive days were
53.2 , 55.7, 54.2 , 52.7, 59.3, 53.8.
(If required, you may use $\sum t = 547.0$, $\sum t^2 = 29957.48.$)
(a) Calculate the mean of the times.
(b) Calculate the standard deviation of the times.
(c) Determine the median of the times.

Example 1.38

Applicants for an assembly job are required to take a test of manual dexterity. The times, in seconds, taken to complete the task by 19 applicants were as follows:
63, 229, 165, 77, 49, 74, 67, 59, 66,102,81, 72, 59, 74, 61, 82, 48, 70, 86.
For these data find
(a) the median,
(b) the upper quartile and lower quartiles.



The quartiles of a data set can also be used to assess whether the data set has any outliers. Outliers are unusual or 'freak' values which differ greatly in magnitude from the majority of the data values. But just how large or small does a value have to be to be an outlier? There is no simple answer to this question, but one 'rule of thumb' developed by the statistician John Tukey is to use 'fences'.

The upper fence is at a value 1.5 times the interquartile range above the upper quartile:

$$\text{Upper fence} = Q_3 + 1.5(Q_3 - Q_1).$$

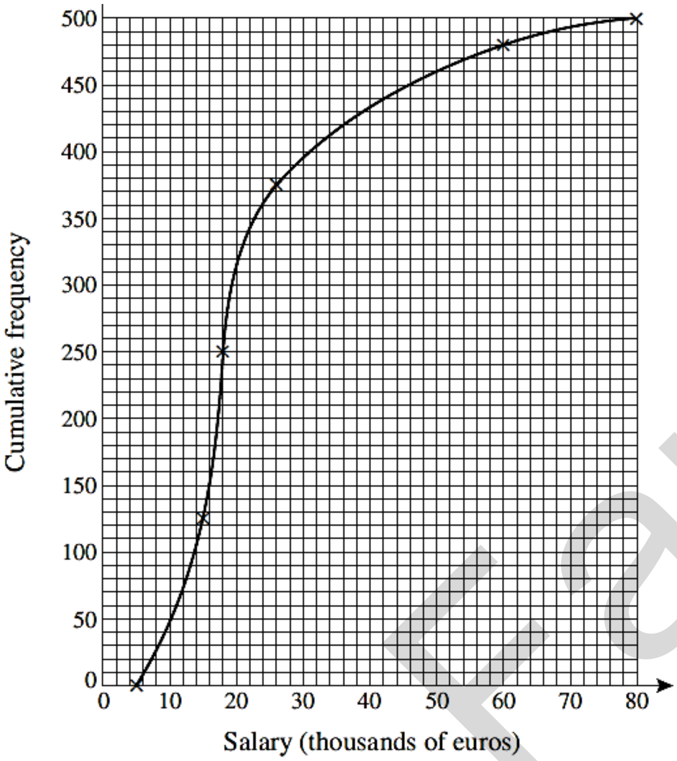
The lower fence is at a value 1.5 times the interquartile range below the lower quartile:

$$\text{Lower fence} = Q_1 - 1.5(Q_3 - Q_1).$$

(c) Identify any outliers in the data.
(d) Illustrate the data by a box and whisker plot. Outliers, if any, should each be denoted by a '*' and should not be included in the whiskers.

Example 1.39

N11/63/Q5



The cumulative frequency graph shows the annual salaries, in thousands of euros, of a random sample of 500 adults with jobs, in France. It has been plotted using grouped data. You may assume that the lowest salary is 5000 euros and the highest salary is 80 000 euros.

(i) On graph paper, draw a box-and-whisker plot to illustrate these salaries. [4]

(ii) Comment on the salaries of the people in this sample. [1]

(iii) An ‘outlier’ is defined as any data value which is more than 1.5 times the interquartile range above the upper quartile, or more than 1.5 times the interquartile range below the lower quartile.

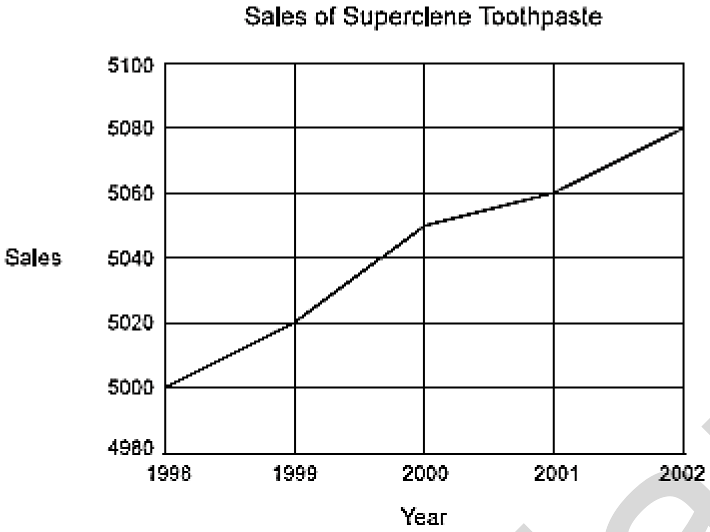
(a) How high must a salary be in order to be classified as an outlier? [3]

(b) Show that none of the salaries is low enough to be classified as an outlier. [1]

GRAPH PAPER

HOMEWORK: REPRESENTATION OF DATA– VARIANT 62

1 (i)



The diagram represents the sales of Superclene toothpaste over the last few years. Give a reason why it is misleading. [1]

(ii) The following data represent the daily ticket sales at a small theatre during three weeks.

52, 73, 34, 85, 62, 79, 89, 50, 45, 83, 84, 91, 85, 84, 87, 44, 86, 41, 35, 73, 86.

(a) Construct a stem-and-leaf diagram to illustrate the data. [3]

(b) Use your diagram to find the median of the data. [1]

Answers: (i) false zero; (ii)(b) 79. **J03/Q1**

---

2 Two cricket teams kept records of the number of runs scored by their teams in 8 matches. The scores are shown in the following table.

Team A	150	220	77	30	298	118	160	57
Team B	166	142	170	93	111	130	148	86

(i) Find the mean and standard deviation of the scores for team A. [2]

The mean and standard deviation for team B are 130.75 and 29.63 respectively.

(ii) State with a reason which team has the more consistent scores. [2]

Answers: (i) 139, 83.1; (ii) team B, smaller standard deviation. **J04/Q1**

3 In a recent survey, 640 people were asked about the length of time each week that they spent watching television. The median time was found to be 20 hours, and the lower and upper quartiles were 15 hours and 35 hours respectively. The least amount of time that anyone spent was 3 hours, and the greatest amount was 60 hours.

(i) On graph paper, show these results using a fully labelled cumulative frequency graph. [3]

(ii) Use your graph to estimate how many people watched more than 50 hours of television each week. [2]

Answer: (ii) Between 40 and 70 if a curve was drawn, or between 60 and 70 if a polygon was drawn. **J04/Q2**

---

4 A random sample of 97 people who own mobile phones was used to collect data on the amount of time they spent per day on their phones. The results are displayed in the table below.

Time spent per day (t minutes)	$0 \leqslant t < 5$	$5 \leqslant t < 10$	$10 \leqslant t < 20$	$20 \leqslant t < 30$	$30 \leqslant t < 40$	$40 \leqslant t < 70$
Number of people	11	20	32	18	10	6

(i) Calculate estimates of the mean and standard deviation of the time spent per day on these mobile phones. [5]

(ii) On graph paper, draw a fully labelled histogram to represent the data. [4]

Answers: (i) 18.4, 13.3; **J03/Q7**
(ii) frequency densities 2.2, 4.0, 3.2, 1.8, 1.0, 0.2 or scaled frequencies usually of 11, 20, 16, 9, 5, 1.

---

5 The following table shows the results of a survey to find the average daily time, in minutes, that a group of schoolchildren spent in internet chat rooms.

Time per day (t minutes)	Frequency
$0 \leqslant t < 10$	2
$10 \leqslant t < 20$	f
$20 \leqslant t < 40$	11
$40 \leqslant t < 80$	4

The mean time was calculated to be 27.5 minutes.

(i) Form an equation involving f and hence show that the total number of children in the survey was 26. [4]

(ii) Find the standard deviation of these times. [2]

Answer: (ii) 16.1. **J05/Q2**

---

6 The following back-to-back stem-and-leaf diagram shows the cholesterol count for a group of 45 people who exercise daily and for another group of 63 who do not exercise. The figures in brackets show the number of people corresponding to each set of leaves.

	People who exercise		People who do not exercise	
(9)	9 8 7 6 4 3 2 2 1	3	1 5 7 7	(4)
(12)	9 8 8 8 7 6 6 5 3 3 2 2	4	2 3 4 4 5 8	(6)
(9)	8 7 7 7 6 5 3 3 1	5	1 2 2 2 3 4 4 5 6 7 8 8 9	(13)
(7)	6 6 6 6 4 3 2	6	1 2 3 3 3 4 5 5 5 7 7 8 9 9	(14)
(3)	8 4 1	7	2 4 5 5 6 6 7 8 8	(9)
(4)	9 5 5 2	8	1 3 3 4 6 7 9 9 9	(9)
(1)	4	9	1 4 5 5 8	(5)
(0)		10	3 3 6	(3)

Key: 2 | 8 | 1 represents a cholesterol count of 8.2 in the group who exercise and 8.1 in the group who do not exercise.

(i) Give one useful feature of a stem-and-leaf diagram. [1]

(ii) Find the median and the quartiles of the cholesterol count for the group who do not exercise. [3]

You are given that the lower quartile, median and upper quartile of the cholesterol count for the group who exercise are 4.25, 5.3 and 6.6 respectively.

(iii) On a single diagram on graph paper, draw two box-and-whisker plots to illustrate the data. [4]

Answer: (ii) 5.4, 6.5, 8.3. **J05/Q4**

---

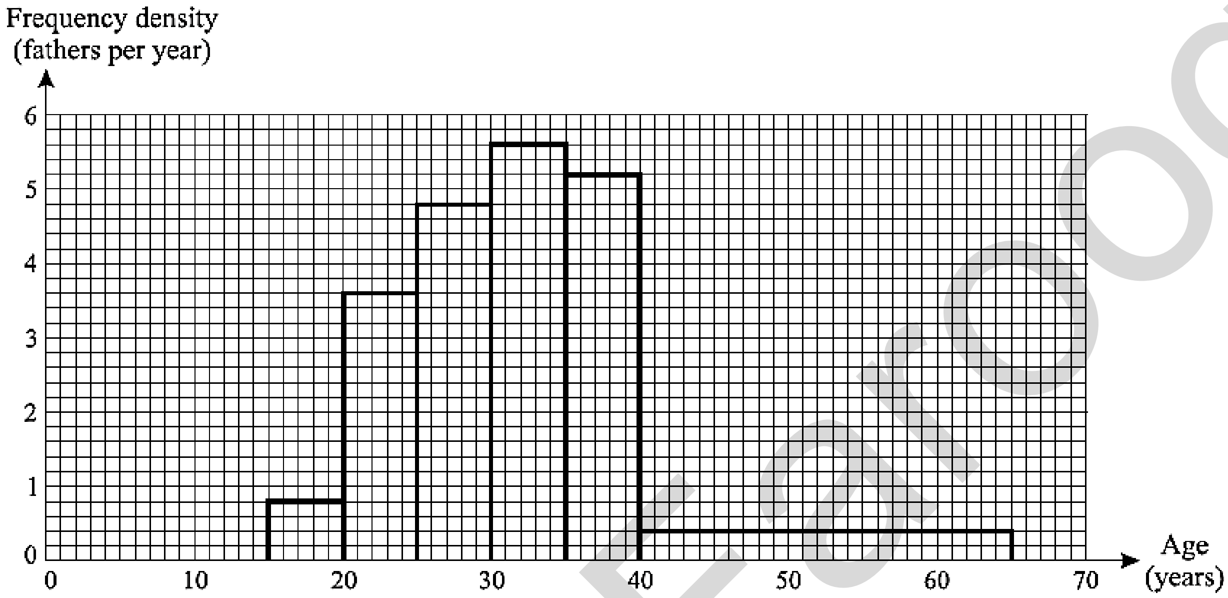
7 The salaries, in thousands of dollars, of 11 people, chosen at random in a certain office, were found to be:

40, 42, 45, 41, 352, 40, 50, 48, 51, 49, 47.

Choose and calculate an appropriate measure of central tendency (mean, mode or median) to summarise these salaries. Explain briefly why the other measures are not suitable. [3]

Answers: median, 47 000, data have an outlier, mode is the lowest etc. **J06/Q1**

8 Each father in a random sample of fathers was asked how old he was when his first child was born. The following histogram represents the information.



(i) What is the modal age group? [1]

(ii) How many fathers were between 25 and 30 years old when their first child was born? [2]

(iii) How many fathers were in the sample? [2]

(iv) Find the probability that a father, chosen at random from the group, was between 25 and 30 years old when his first child was born, given that he was older than 25 years. [2]

Answers: (i) 30-35 years; **(ii)** 24; **(iii)** 110; **(iv)** 0.273.

J06/Q5

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9 The length of time, t minutes, taken to do the crossword in a certain newspaper was observed on 12 occasions. The results are summarised below.

$$\Sigma(t-35) = -15 \qquad \Sigma(t-35)^2 = 82.23$$

Calculate the mean and standard deviation of these times taken to do the crossword. [4]

J07/Q1

Answers: 33.8 minutes, 2.3 minutes

10 The lengths of time in minutes to swim a certain distance by the members of a class of twelve 9-year-olds and by the members of a class of eight 16-year-olds are shown below.

9-year-olds:	13.0	16.1	16.0	14.4	15.9	15.1	14.2	13.7	16.7	16.4	15.0	13.2
16-year-olds:	14.8	13.0	11.4	11.7	16.5	13.7	12.8	12.9				

(i) Draw a back-to-back stem-and-leaf diagram to represent the information above. [4]

(ii) A new pupil joined the 16-year-old class and swam the distance. The mean time for the class of nine pupils was now 13.6 minutes. Find the new pupil's time to swim the distance. [3]

Answer: (ii) 15.6 minutes. **J07/Q4**

---

11 The stem-and-leaf diagram below represents data collected for the number of hits on an internet site on each day in March 2007. There is one missing value, denoted by x.

0	0 1 5 6	(4)
1	1 3 5 6 6 8	(6)
2	1 1 2 3 4 4 4 8 9	(9)
3	1 2 2 2 x 8 9	(7)
4	2 5 6 7 9	(5)

Key: 1 | 5 represents 15 hits

(i) Find the median and lower quartile for the number of hits each day. [2]

(ii) The interquartile range is 19. Find the value of x. [2]

Answers: (i) 24, 16; (ii) 5. **J08/Q1**

---

12 As part of a data collection exercise, members of a certain school year group were asked how long they spent on their Mathematics homework during one particular week. The times are given to the nearest 0.1 hour. The results are displayed in the following table.

Time spent (t hours)	$0.1 \leqslant t \leqslant 0.5$	$0.6 \leqslant t \leqslant 1.0$	$1.1 \leqslant t \leqslant 2.0$	$2.1 \leqslant t \leqslant 3.0$	$3.1 \leqslant t \leqslant 4.5$
Frequency	11	15	18	30	21

(i) Draw, on graph paper, a histogram to illustrate this information. [5]

(ii) Calculate an estimate of the mean time spent on their Mathematics homework by members of this year group. [3]

Answer: (ii) 2.1 hours. **J08/Q5**

13 During January the numbers of people entering a store during the first hour after opening were as follows.

Time after opening, x minutes	Frequency	Cumulative frequency
$0 < x \leqslant 10$	210	210
$10 < x \leqslant 20$	134	344
$20 < x \leqslant 30$	78	422
$30 < x \leqslant 40$	72	a
$40 < x \leqslant 60$	b	540

(i) Find the values of a and b. [2]

(ii) Draw a cumulative frequency graph to represent this information. Take a scale of 2 cm for 10 minutes on the horizontal axis and 2 cm for 50 people on the vertical axis. [4]

(iii) Use your graph to estimate the median time after opening that people entered the store. [2]

(iv) Calculate estimates of the mean, m minutes, and standard deviation, s minutes, of the time after opening that people entered the store. [4]

(v) Use your graph to estimate the number of people entering the store between $(m - \frac{1}{2}s)$ and $(m + \frac{1}{2}s)$ minutes after opening. [2]

Answers: (i) 494, 46; (iii) 13.5 to 14.6 minutes; (iv) 18.2 minutes, 14.2 minutes; (v) 155 to 170 people.

J09/Q6

---

14 The times in minutes for seven students to become proficient at a new computer game were measured. The results are shown below.

15 10 48 10 19 14 16

(i) Find the mean and standard deviation of these times. [2]

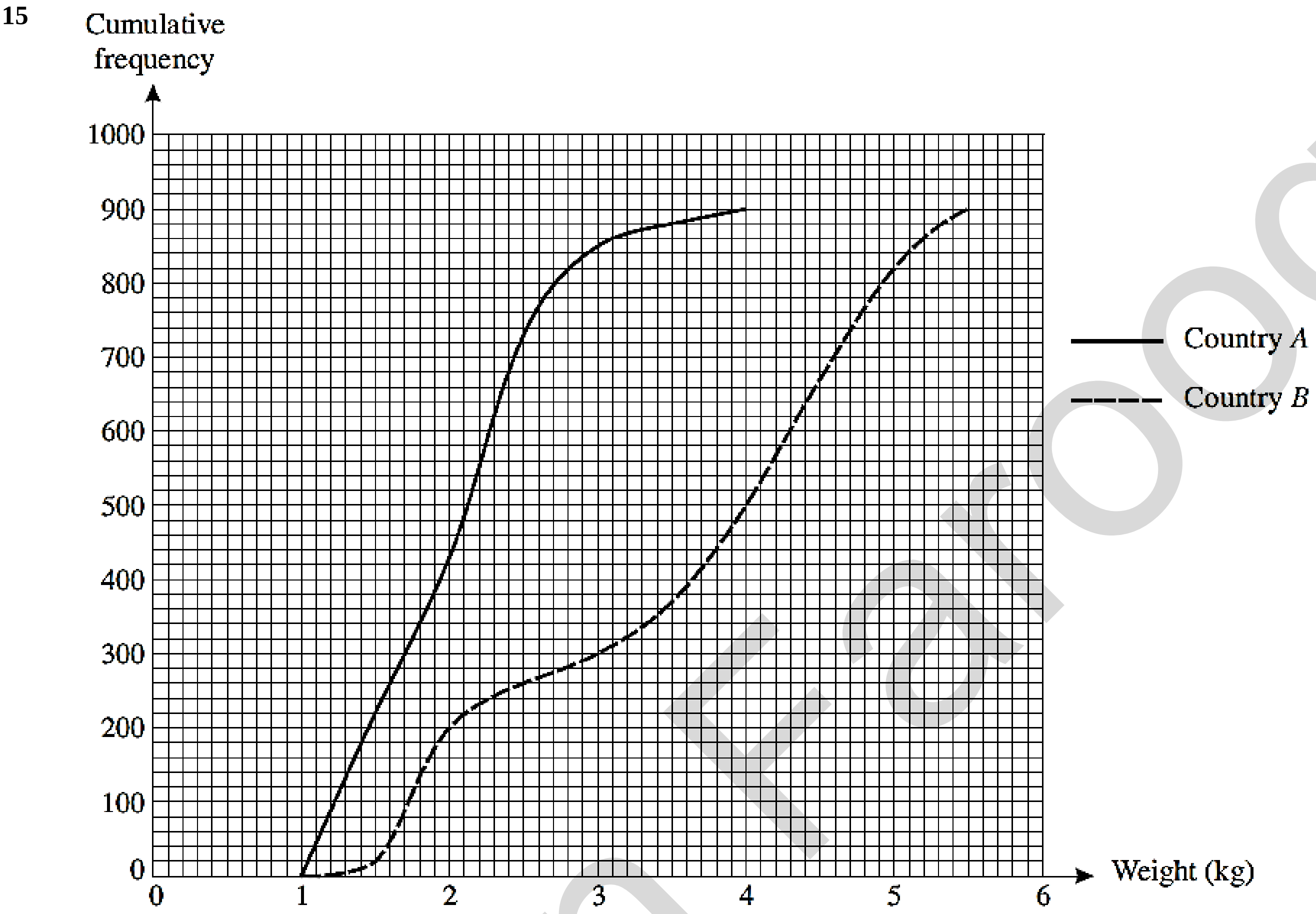
(ii) State which of the mean, median or mode you consider would be most appropriate to use as a measure of central tendency to represent the data in this case. [1]

(iii) For each of the two measures of average you did not choose in part **(ii)**, give a reason why you consider it inappropriate. [2]

Answers: (i) 18.9, 12.3; (ii) median; (iii) mode, 10, inappropriate because it is the smallest number; mean inappropriate because it is affected by the outlier, 48

J10/62/Q1

15



The birth weights of random samples of 900 babies born in country *A* and 900 babies born in country *B* are illustrated in the cumulative frequency graphs. Use suitable data from these graphs to compare the central tendency and spread of the birth weights of the two sets of babies. [6]

Answers: median *A* = 2.0, median *B* = 3.8, interquartile range *A* = 0.9, interquartile range *B* = 2.3, country *B* has heavier babies and greater spread than country *B*. **J10/62/Q3**

16 The weights in kilograms of two groups of 17-year-old males from country P and country Q are displayed in the following back-to-back stem-and-leaf diagram. In the third row of the diagram, ... 4 | 7 | 1 ... denotes weights of 74 kg for a male in country P and 71 kg for a male in country Q.

Country P		Country Q
	5	1 5
	6	2 3 4 8
9 8 7 6 4	7	1 3 4 5 6 7 7 8 8 9
8 8 6 6 5 3	8	2 3 6 7 7 8 8
9 7 7 6 5 5 5 4 2	9	0 2 2 4
5 4 4 3 1	10	4 5

(i) Find the median and quartile weights for country Q. [3]

(ii) You are given that the lower quartile, median and upper quartile for country P are 84, 94 and 98 kg respectively. On a single diagram on graph paper, draw two box-and-whisker plots of the data. [4]

(iii) Make two comments on the weights of the two groups. [2]

Answers: (i) LQ 72 or 73 or 71.5, Median 78, UQ 88 or 87.75;
(iii) 'people heavier in P than Q', or 'weights more spread out in Q than P', or 'P is negatively skewed, Q is positively skewed or more symmetrical'.

N02/Q7

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17 A computer can generate random numbers which are either 0 or 2. On a particular occasion, it generates a set of numbers which consists of 23 zeros and 17 twos. Find the mean and variance of this set of 40 numbers. [4]

Answers: 0.850, 0.978.

N03/Q1

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18 The floor areas, $x\,\text{m}^2$, of 20 factories are as follows.

150	350	450	578	595	644	722	798	802	904
1000	1330	1533	1561	1778	1960	2167	2330	2433	3231

Represent these data by a histogram on graph paper, using intervals

$$0 \leqslant x < 500,\ 500 \leqslant x < 1000,\ 1000 \leqslant x < 2000,\ 2000 \leqslant x < 3000,\ 3000 \leqslant x < 4000.$$ [4]

N03/Q2

19 The lengths of cars travelling on a car ferry are noted. The data are summarised in the following table.

Length of car (x metres)	Frequency	Frequency density
$2.80 \leqslant x < 3.00$	17	85
$3.00 \leqslant x < 3.10$	24	240
$3.10 \leqslant x < 3.20$	19	190
$3.20 \leqslant x < 3.40$	8	a

(i) Find the value of a. [1]

(ii) Draw a histogram on graph paper to represent the data. [3]

(iii) Find the probability that a randomly chosen car on the ferry is less than 3.20 m in length. [2]

Answers: (i) 40; (iii) $\frac{60}{68}$ or 0.882. **N04/Q2**

---

20 The ages, x years, of 18 people attending an evening class are summarised by the following totals: $\Sigma x = 745, \Sigma x^2 = 33\,951$.

(i) Calculate the mean and standard deviation of the ages of this group of people. [3]

(ii) One person leaves the group and the mean age of the remaining 17 people is exactly 41 years. Find the age of the person who left and the standard deviation of the ages of the remaining 17 people. [4]

Answers: (i) 13.2; (ii) 48, 13.4. **N04/Q4**

---

21 A study of the ages of car drivers in a certain country produced the results shown in the table.

Percentage of drivers in each age group

	Young	Middle-aged	Elderly
Males	40	35	25
Females	20	70	10

Illustrate these results diagrammatically. [4]

N05/Q1

22 A group of 10 married couples and 3 single men found that the mean age $\bar{x}_w$ of the 10 women was 41.2 years and the standard deviation of the women's ages was 15.1 years. For the 13 men, the mean age $\bar{x}_m$ was 46.3 years and the standard deviation was 12.7 years.

(i) Find the mean age of the whole group of 23 people. [2]

(ii) The individual women's ages are denoted by x_w and the individual men's ages by x_m. By first finding Σx_w^2 and Σx_m^2, find the standard deviation for the whole group. [5]

Answers: (i) 44.1; (ii) 14.0. **N05/Q4**

---

23 The weights of 30 children in a class, to the nearest kilogram, were as follows.

50	45	61	53	55	47	52	49	46	51
60	52	54	47	57	59	42	46	51	53
56	48	50	51	44	52	49	58	55	45

Construct a grouped frequency table for these data such that there are five equal class intervals with the first class having a lower boundary of 41.5 kg and the fifth class having an upper boundary of 61.5 kg. [4]

N06/Q1

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24 In a survey, people were asked how long they took to travel to and from work, on average. The median time was 3 hours 36 minutes, the upper quartile was 4 hours 42 minutes and the interquartile range was 3 hours 48 minutes. The longest time taken was 5 hours 12 minutes and the shortest time was 30 minutes.

(i) Find the lower quartile. [2]

(ii) Represent the information by a box-and-whisker plot, using a scale of 2 cm to represent 60 minutes. [4]

Answers: (i) 54 minutes. **N06/Q3**

---

25 A summary of 24 observations of x gave the following information:

$$\Sigma(x-a) = -73.2 \quad \text{and} \quad \Sigma(x-a)^2 = 2115.$$

The mean of these values of x is 8.95.

(i) Find the value of the constant a. [2]

(ii) Find the standard deviation of these values of x. [2]

Answers: (i) 12; (ii) 8.88. **N07/Q1**

26 The arrival times of 204 trains were noted and the number of minutes, t, that each train was late was recorded. The results are summarised in the table.

Number of minutes late (t)	$-2 \leqslant t < 0$	$0 \leqslant t < 2$	$2 \leqslant t < 4$	$4 \leqslant t < 6$	$6 \leqslant t < 10$
Number of trains	43	51	69	22	19

(i) Explain what $-2 \leqslant t < 0$ means about the arrival times of trains. [1]

(ii) Draw a cumulative frequency graph, and from it estimate the median and the interquartile range of the number of minutes late of these trains. [7]

Answers: (ii) median rounding to 2.1 – 2.4 minutes, interquartile range rounding to 3.2 – 3.6 minutes. **N07/Q5**

---

27 Rachel measured the lengths in millimetres of some of the leaves on a tree. Her results are recorded below.

32 35 45 37 38 44 33 39 36 45

Find the mean and standard deviation of the lengths of these leaves. [3]

Answers: 38.4 mm, 4.57 mm. **N08/Q1**

---

28 The pulse rates, in beats per minute, of a random sample of 15 small animals are shown in the following table.

115	120	158	132	125
104	142	160	145	104
162	117	109	124	134

(i) Draw a stem-and-leaf diagram to represent the data. [3]

(ii) Find the median and the quartiles. [2]

(iii) On graph paper, using a scale of 2 cm to represent 10 beats per minute, draw a box-and-whisker plot of the data. [3]

Answers: (ii) 125, 115, 145. **N08/Q5**

29 The following table gives the marks, out of 75, in a pure mathematics examination taken by 234 students.

Marks	1–20	21–30	31–40	41–50	51–60	61–75
Frequency	40	34	56	54	29	21

(i) Draw a histogram on graph paper to represent these results. [5]

(ii) Calculate estimates of the mean mark and the standard deviation. [4]

N09/62/Q6

Answers: (ii) 37.5, 16.9.

---

30 Esme noted the test marks, x, of 16 people in a class. She found that $\Sigma x = 824$ and that the standard deviation of x was 6.5.

(i) Calculate $\Sigma(x - 50)$ and $\Sigma(x - 50)^2$. [3]

(ii) One person did the test later and her mark was 72. Calculate the new mean and standard deviation of the marks of all 17 people. [3]

N10/62/Q2

Answers: (i) 24, 712; (ii) 52.7, 7.94.

---

31 The weights in kilograms of 11 bags of sugar and 7 bags of flour are as follows.

Sugar:	1.961	1.983	2.008	2.014	1.968	1.994	2.011	2.017	1.977	1.984	1.989
Flour:	1.945	1.962	1.949	1.977	1.964	1.941	1.953				

(i) Represent this information on a back-to-back stem-and-leaf diagram with sugar on the left-hand side. [4]

(ii) Find the median and interquartile range of the weights of the bags of sugar. [3]

N10/62/Q4

Answers: (ii) 1.989 kg, 0.034 kg.

32 32 teams enter for a knockout competition, in which each match results in one team winning and the other team losing. After each match the winning team goes on to the next round, and the losing team takes no further part in the competition. Thus 16 teams play in the second round, 8 teams play in the third round, and so on, until 2 teams play in the final round.

(i) How many teams play in only 1 match? [1]

(ii) How many teams play in exactly 2 matches? [1]

(iii) Draw up a frequency table for the numbers of matches which the teams play. [3]

(iv) Calculate the mean and variance of the numbers of matches which the teams play. [4]

Answers: (i) 16; (ii) 8; (iii) matches 1, 2, 3, 4, 5 frequencies 16, 8, 4, 2, 2; (iv) mean 1.94, variance 1.43. **J06/Q6**

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33 A sample of 36 data values, x, gave $\Sigma(x-45) = -148$ and $\Sigma(x-45)^2 = 3089$.

(i) Find the mean and standard deviation of the 36 values. [3]

(ii) One extra data value of 29 was added to the sample. Find the standard deviation of all 37 values. [4]

Answers: (i) 40.9, 8.30; (ii) 8.41. **J11/62/Q3**

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34 A hotel has 90 rooms. The table summarises information about the number of rooms occupied each day for a period of 200 days.

Number of rooms occupied	1 – 20	21 – 40	41 – 50	51 – 60	61 – 70	71 – 90
Frequency	10	32	62	50	28	18

(i) Draw a cumulative frequency graph on graph paper to illustrate this information. [4]

(ii) Estimate the number of days when over 30 rooms were occupied. [2]

(iii) On 75% of the days at most n rooms were occupied. Estimate the value of n. [2]

Answers: (ii) 174 – 180; (iii) 59 or 60. **J11/62/Q5**

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35 The ages, x years, of 150 cars are summarised by $\Sigma x = 645$ and $\Sigma x^2 = 8287.5$. Find $\Sigma(x-\bar{x})^2$, where $\bar{x}$ denotes the mean of x. [4]

Answer: 5514 **J12/62/Q1**

36 The back-to-back stem-and-leaf diagram shows the values taken by two variables A and B.

	A		B	
(3)	3 1 0	15	1 3 3 5	(4)
(2)	4 1	16	2 2 3 4 4 5 7 7 7 8	(10)
(3)	8 3 3	17	0 1 3 3 3 4 6 6 7 9 9	(11)
(12)	9 8 8 6 5 5 4 3 2 1 1 0	18	2 4 7	(3)
(8)	9 9 8 8 6 5 4 2	19	1 5	(2)
(5)	9 8 7 1 0	20	4	(1)

Key: 4 | 16 | 7 means $A = 0.164$ and $B = 0.167$.

(i) Find the median and the interquartile range for variable A. [3]

(ii) You are given that, for variable B, the median is 0.171, the upper quartile is 0.179 and the lower quartile is 0.164. Draw box-and-whisker plots for A and B in a single diagram on graph paper. [3]

Answers: (i) Median = 0.186, IQ range = 0.019 **J12/62/Q4**

---

37 A summary of the speeds, x kilometres per hour, of 22 cars passing a certain point gave the following information:

$$\Sigma(x-50) = 81.4 \qquad \text{and} \qquad \Sigma(x-50)^2 = 671.0.$$

Find the variance of the speeds and hence find the value of Σx^2. [4]

Answers: 16.81, 63811 **J13/62/Q2**

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38 The following are the annual amounts of money spent on clothes, to the nearest \$10, by 27 people.

10	40	60	80	100	130	140	140	140
150	150	150	160	160	160	160	170	180
180	200	210	250	270	280	310	450	570

(i) Construct a stem-and-leaf diagram for the data. [3]

(ii) Find the median and the interquartile range of the data. [3]

An 'outlier' is defined as any data value which is more than 1.5 times the interquartile range above the upper quartile, or more than 1.5 times the interquartile range below the lower quartile.

(iii) List the outliers. [3]

Answers: IQ R 70, outliers 10, 450, 570 **J13/62/Q5**

39 The times taken by 57 athletes to run 100 metres are summarised in the following cumulative frequency table.

Time (seconds)	<10.0	<10.5	<11.0	<12.0	<12.5	<13.5
Cumulative frequency	0	4	10	40	49	57

(i) State how many athletes ran 100 metres in a time between 10.5 and 11.0 seconds. [1]

(ii) Draw a histogram on graph paper to represent the times taken by these athletes to run 100 metres. [4]

(iii) Calculate estimates of the mean and variance of the times taken by these athletes. [4]

Answers: 6, 11.7, 0.547 **J14/62/Q6**

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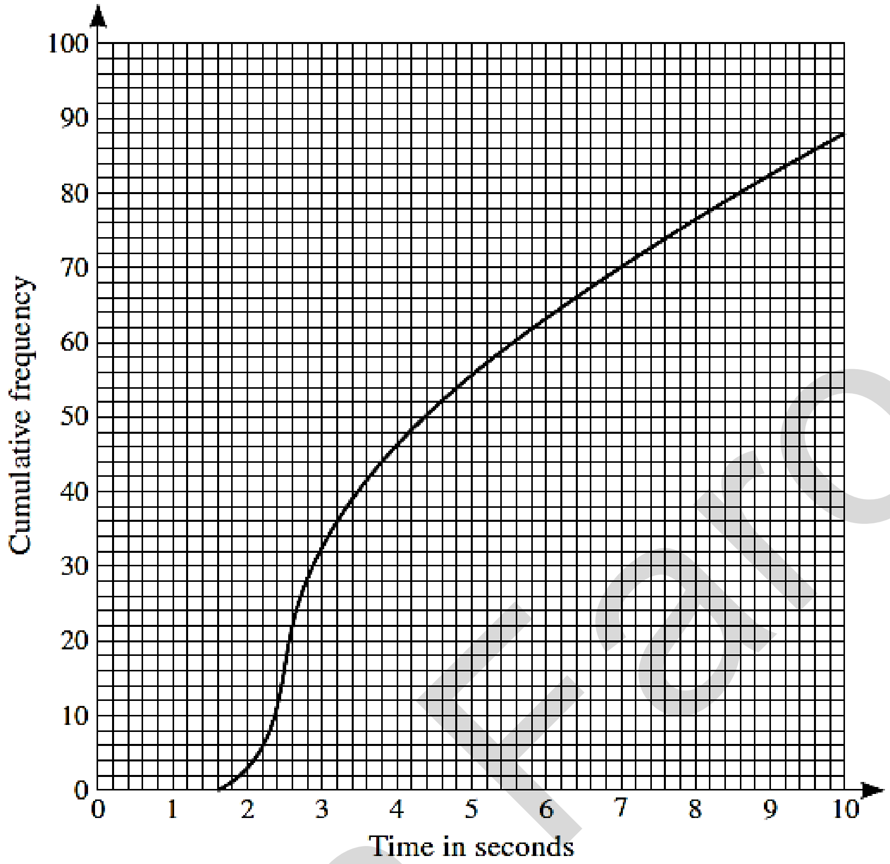
40 120 people were asked to read an article in a newspaper. The times taken, to the nearest second, by the people to read the article are summarised in the following table.

Time (seconds)	1 – 25	26 – 35	36 – 45	46 – 55	56 – 90
Number of people	4	24	38	34	20

Calculate estimates of the mean and standard deviation of the reading times. [5]

Answers: 45.8, 14.9 **J15/62/Q2**

41



In an open-plan office there are 88 computers. The times taken by these 88 computers to access a particular web page are represented in the cumulative frequency diagram.

(i) On graph paper draw a box-and-whisker plot to summarise this information. [4]

An 'outlier' is defined as any data value which is more than 1.5 times the interquartile range above the upper quartile, or more than 1.5 times the interquartile range below the lower quartile.

(ii) Show that there are no outliers. [2]

Answers: (i) LQ 2.6, median 3.8, UQ 6.4. (ii) lower fence –3.1, upper fence 12.1 no values exist hence no outliers **J15/62/Q3**

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42 The following are the times, in minutes, taken by 11 runners to complete a 10 km run.

48.3 55.2 59.9 67.7 60.5 75.6 62.5 57.4 53.4 49.2 64.1

Find the mean and standard deviation of these times. [3]

Answers: (i) 59.4; (ii) 7.68 **N11/62/Q1**

43 The weights of 220 sausages are summarised in the following table.

Weight (grams)	<20	<30	<40	<45	<50	<60	<70
Cumulative frequency	0	20	50	100	160	210	220

(i) State which interval the median weight lies in. [1]

(ii) Find the smallest possible value and the largest possible value for the interquartile range. [2]

(iii) State how many sausages weighed between 50 g and 60 g. [1]

(iv) On graph paper, draw a histogram to represent the weights of the sausages. [4]

Answers: (i) 45-50g; (ii) smallest IQR 5 largest 10; (iii) 50 **N11/62/Q4**

---

44 The table summarises the times that 112 people took to travel to work on a particular day.

Time to travel to work (t minutes)	$0 < t \leqslant 10$	$10 < t \leqslant 15$	$15 < t \leqslant 20$	$20 < t \leqslant 25$	$25 < t \leqslant 40$	$40 < t \leqslant 60$
Frequency	19	12	28	22	18	13

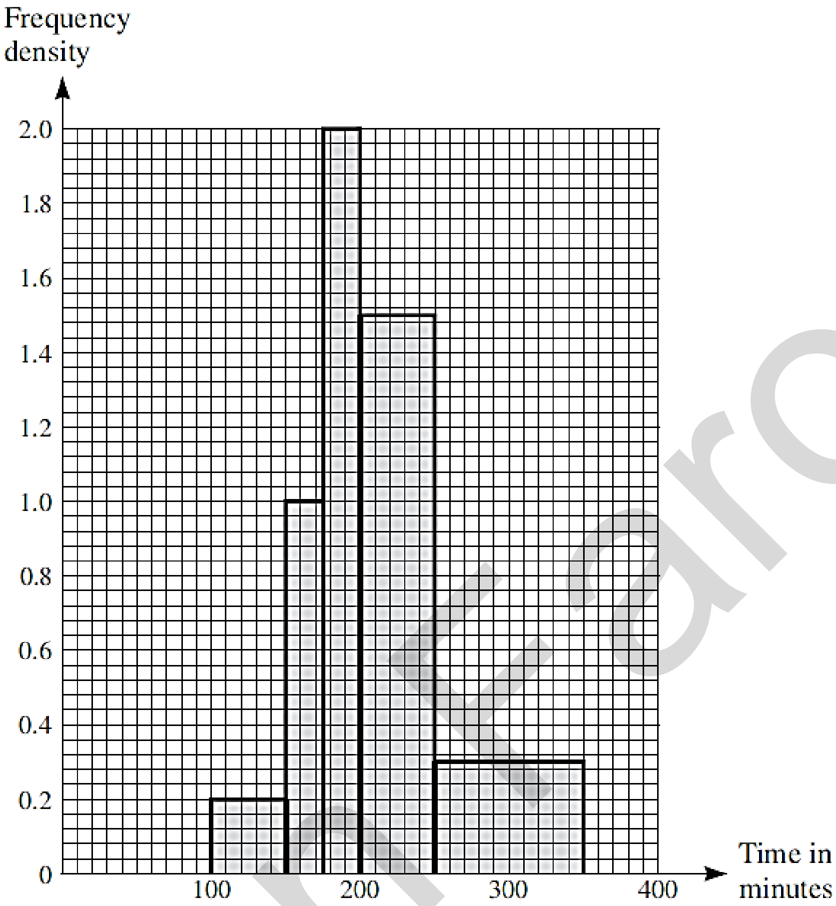
(i) State which time interval in the table contains the median and which time interval contains the upper quartile. [2]

(ii) On graph paper, draw a histogram to represent the data. [4]

(iii) Calculate an estimate of the mean time to travel to work. [2]

Answers: (i) 15-20 mins, 25-40 mins; (iii) 22.0 minutes. **N12/62/Q3**

45 The following histogram summarises the times, in minutes, taken by 190 people to complete a race.



(i) Show that 75 people took between 200 and 250 minutes to complete the race. [1]

(ii) Calculate estimates of the mean and standard deviation of the times of the 190 people. [6]

(iii) Explain why your answers to part **(ii)** are estimates. [1]

Answers: 213, 46.5

N13/62/Q4

46 On a certain day in spring, the heights of 200 daffodils are measured, correct to the nearest centimetre. The frequency distribution is given below.

Height (cm)	4 – 10	11 – 15	16 – 20	21 – 25	26 – 30
Frequency	22	32	78	40	28

(i) Draw a cumulative frequency graph to illustrate the data. [4]

(ii) 28% of these daffodils are of height h cm or more. Estimate h. [2]

(iii) You are given that the estimate of the mean height of these daffodils, calculated from the table, is 18.39 cm. Calculate an estimate of the standard deviation of the heights of these daffodils. [3]

Answers: (ii) a single value between 21 and 23 cm, 6.01

N14/62/Q6

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47 For n values of the variable x, it is given that $\Sigma(x - 100) = 216$ and $\Sigma x = 2416$. Find the value of n. [3]

Answer: 22

N15/62/Q1

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48 The weights, in kilograms, of the 15 rugby players in each of two teams, A and B, are shown below.

Team A	97	98	104	84	100	109	115	99	122	82	116	96	84	107	91
Team B	75	79	94	101	96	77	111	108	83	84	86	115	82	113	95

(i) Represent the data by drawing a back-to-back stem-and-leaf diagram with team A on the left-hand side of the diagram and team B on the right-hand side. [4]

(ii) Find the interquartile range of the weights of the players in team A. [2]

(iii) A new player joins team B as a substitute. The mean weight of the 16 players in team B is now 93.9 kg. Find the weight of the new player. [3]

Answers: (ii) 18, (iii) 103

N15/62/Q5

HOMEWORK:REPRESENTATION OF DATA VARIANT 61 AND 63

1 The table summarises the lengths in centimetres of 104 dragonflies.

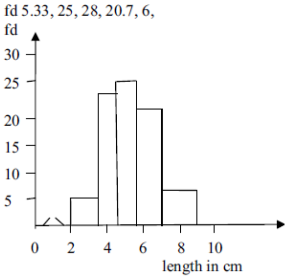
Length (cm)	2.0 – 3.5	3.5 – 4.5	4.5 – 5.5	5.5 – 7.0	7.0 – 9.0
Frequency	8	25	28	31	12

(i) State which class contains the upper quartile. [1]

(ii) Draw a histogram, on graph paper, to represent the data. [4]

61/J15/2

Answer: a) 5.5-7.0 b)



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2 The table shows the mean and standard deviation of the weights of some turkeys and geese.

	Number of birds	Mean (kg)	Standard deviation (kg)
Turkeys	9	7.1	1.45
Geese	18	5.2	0.96

(i) Find the mean weight of the 27 birds. [2]

(ii) The weights of individual turkeys are denoted by x_t kg and the weights of individual geese by x_g kg. By first finding Σx_t^2 and Σx_g^2, find the standard deviation of the weights of all 27 birds. [5]

Answer: i) 5.83 ii) 1.46 **61/J15/5**

3 Seventy samples of fertiliser were collected and the nitrogen content was measured for each sample. The cumulative frequency distribution is shown in the table below.

Nitrogen content	$\leqslant 3.5$	$\leqslant 3.8$	$\leqslant 4.0$	$\leqslant 4.2$	$\leqslant 4.5$	$\leqslant 4.8$
Cumulative frequency	0	6	18	41	62	70

(i) On graph paper draw a cumulative frequency graph to represent the data. [3]

(ii) Estimate the percentage of samples with a nitrogen content greater than 4.4. [2]

(iii) Estimate the median. [1]

(iv) Construct the frequency table for these results and draw a histogram on graph paper. [5]

Answer: ii) 21.4% iii)41.5 **63/J15/6**

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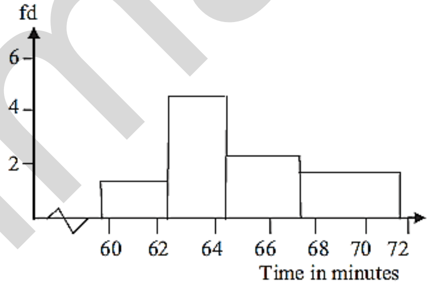
4 Robert has a part-time job delivering newspapers. On a number of days he noted the time, correct to the nearest minute, that it took him to do his job. Robert used his results to draw up the following table; two of the values in the table are denoted by a and b.

Time (t minutes)	60 – 62	63 – 64	65 – 67	68 – 71
Frequency (number of days)	3	9	6	b
Frequency density	1	a	2	1.5

(i) Find the values of a and b. [3]

(ii) On graph paper, draw a histogram to represent Robert's times. [3]

61/N15/3



Answer: i) 4.5, 6 ii)

---

5 (a) Amy measured her pulse rate while resting, x beats per minute, at the same time each day on 30 days. The results are summarised below.

$$\Sigma(x-80) = -147 \qquad \Sigma(x-80)^2 = 952$$

Find the mean and standard deviation of Amy's pulse rate. [4]

Answer: i)75.1, 2.78 **61/N15/4**

---

6 The time taken, t hours, to deliver letters on a particular route each day is measured on 250 working days. The mean time taken is 2.8 hours. Given that $\Sigma(t - 2.5)^2 = 96.1$, find the standard deviation of the times taken. [3]

Answer: 0.543 **63/N15/1**

---

7 The heights to the nearest metre of 134 office buildings in a certain city are summarised in the table below.

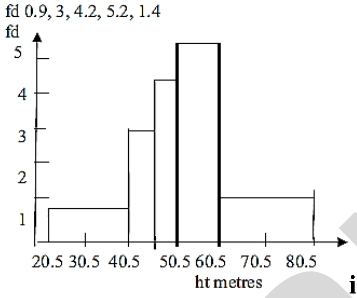
Height (m)	21 – 40	41 – 45	46 – 50	51 – 60	61 – 80
Frequency	18	15	21	52	28

(i) Draw a histogram on graph paper to illustrate the data. [4]

(ii) Calculate estimates of the mean and standard deviation of these heights. [5]

63/N15/6

Answer: i)



ii) 12.2

---

8 A traffic camera measured the speeds, x kilometres per hour, of 8 cars travelling along a certain street, with the following results.

62.7 59.6 64.2 61.5 68.3 66.9 62.0 62.3

(i) Find $\Sigma(x - 62)$. [1]

(ii) Find $\Sigma(x - 62)^2$. [1]

(iii) Find the mean and variance of the speeds of the 8 cars. [3]

Answer: i) 11.5 ii) 75.13 **63/N14/2**

9 A random sample of 25 people recorded the number of glasses of water they drank in a particular week. The results are shown below.

23	19	32	14	25
22	26	36	45	42
47	28	17	38	15
46	18	26	22	41
19	21	28	24	30

(i) Draw a stem-and-leaf diagram to represent the data. [3]

(ii) On graph paper draw a box-and-whisker plot to represent the data. [5]

Answer: **63/N14/4**

---

10 On a certain day in spring, the heights of 200 daffodils are measured, correct to the nearest centimetre. The frequency distribution is given below.

Height (cm)	4 – 10	11 – 15	16 – 20	21 – 25	26 – 30
Frequency	22	32	78	40	28

(i) Draw a cumulative frequency graph to illustrate the data. [4]

(ii) 28% of these daffodils are of height h cm or more. Estimate h. [2]

(iii) You are given that the estimate of the mean height of these daffodils, calculated from the table, is 18.39 cm. Calculate an estimate of the standard deviation of the heights of these daffodils. [3]

Answer: ii) 22.5 iii) 6.01 **62/N14/6**

---

11 Find the mean and variance of the following data. [3]

$$5 \quad -2 \quad 12 \quad 7 \quad -3 \quad 2 \quad -6 \quad 4 \quad 0 \quad 8$$

Answer: 2.7 and 27.8 **61/N14/1**

12 The following back-to-back stem-and-leaf diagram shows the times to load an application on 61 smartphones of type *A* and 43 smartphones of type *B*.

	Type *A*		Type *B*	
(7)	9 7 6 6 4 3 3	2	1 3 5 8	(4)
(7)	5 5 4 4 2 2 2	3	0 4 4 5 6 6 6 6 7 8 8 9	(12)
(13)	9 9 8 8 8 7 6 6 4 3 2 2 0	4	0 1 1 2 3 6 8 8 9 9	(10)
(9)	6 5 5 4 3 2 1 1 0	5	2 5 6 6 9	(5)
(4)	9 7 3 0	6	1 3 8 9	(4)
(6)	8 7 4 4 1 0	7	5 7	(2)
(10)	7 6 6 6 5 3 3 2 1 0	8	1 2 4 4	(4)
(5)	8 6 5 5 5	9	0 6	(2)

Key: 3 | 2 | 1 means 0.23 seconds for type *A* and 0.21 seconds for type *B*.

(i) Find the median and quartiles for smartphones of type *A*. [3]

You are given that the median, lower quartile and upper quartile for smartphones of type *B* are 0.46 seconds, 0.36 seconds and 0.63 seconds respectively.

(ii) Represent the data by drawing a pair of box-and-whisker plots in a single diagram on graph paper. [3]

(iii) Compare the loading times for these two types of smartphone. [1]

Answer: M=0.5, LQ= 0.41 UQ=0.79 iii) smartphone B is quicker, slightly less variable

12 a Some adults and some children each tried to estimate, without using a watch, the number of seconds that had elapsed in a fixed time-interval. Their estimates are shown below.

Adults:	55	58	67	74	63	61	63	71	56	53	54	78	73	64	62
Children:	86	95	89	72	61	84	77	92	81	54	43	68	62	67	83

(i) Draw a back-to-back stem-and-leaf diagram to represent the data. [3]

(ii) Make two comparisons between the estimates of the adults and the children. [2]

Answer: ii) Spread and symmetry statements **63/J14/1**

13 The heights, x cm, of a group of 28 people were measured. The mean height was found to be 172.6 cm and the standard deviation was found to be 4.58 cm. A person whose height was 161.8 cm left the group.

(i) Find the mean height of the remaining group of 27 people. [2]

(ii) Find Σx^2 for the original group of 28 people. Hence find the standard deviation of the heights of the remaining group of 27 people. [4]

Answer: i) 1.73 ii) 835000,4.16 **63/J14/4**

---

14 A typing test is taken by 111 people. The numbers of typing errors they make in the test are summarised in the table below.

Number of typing errors	1 – 5	6 – 20	21 – 35	36 – 60	61 – 80
Frequency	24	9	21	15	42

(i) Draw a histogram on graph paper to represent this information. [5]

(ii) Calculate an estimate of the mean number of typing errors for these 111 people. [3]

(iii) State which class contains the lower quartile and which class contains the upper quartile. Hence find the least possible value of the interquartile range. [3]

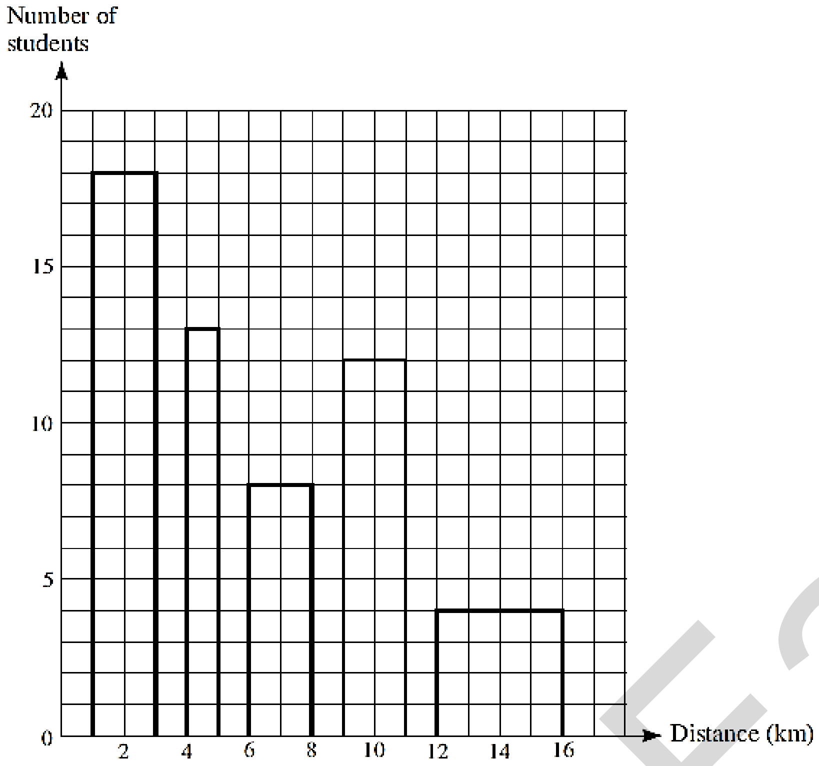
Answer: ii) 40.2 iii) LQ in 6-20 UQ in 61-80 Least IQ=41 **61/J14/7**

---

15 The distance of a student's home from college, correct to the nearest kilometre, was recorded for each of 55 students. The distances are summarised in the following table.

Distance from college (km)	1 – 3	4 – 5	6 – 8	9 – 11	12 – 16
Number of students	18	13	8	12	4

Dominic is asked to draw a histogram to illustrate the data. Dominic's diagram is shown below.



Give two reasons why this is not a correct histogram

63/N13/1

---

16 Barry weighs 20 oranges and 25 lemons. For the oranges, the mean weight is 220 g and the standard deviation is 32 g. For the lemons, the mean weight is 118 g and the standard deviation is 12 g.

(i) Find the mean weight of the 45 fruits. [2]

(ii) The individual weights of the oranges in grams are denoted by x_o, and the individual weights of the lemons in grams are denoted by x_l. By first finding Σx_o^2 and Σx_l^2, find the variance of the weights of the 45 fruits. [5]

Answer: i) 163 ii)20 **63/N13/4**

---

17 Swati measured the lengths, x cm, of 18 stick insects and found that $\Sigma x^2 = 967$. Given that the mean length is $\frac{58}{9}$ cm, find the values of $\Sigma(x-5)$ and $\Sigma(x-5)^2$. [5]

Answer: 26 and 257 **61/N13/3**

18 The following are the house prices in thousands of dollars, arranged in ascending order, for 51 houses from a certain area.

253	270	310	354	386	428	433	468	472	477	485	520	520	524	526	531	535
536	538	541	543	546	548	549	551	554	572	583	590	605	614	638	649	652
666	670	682	684	690	710	725	726	731	734	745	760	800	854	863	957	986

(i) Draw a box-and-whisker plot to represent the data. [4]

An expensive house is defined as a house which has a price that is more than 1.5 times the interquartile range above the upper quartile.

(ii) For the above data, give the prices of the expensive houses. [2]

(iii) Give one disadvantage of using a box-and-whisker plot rather than a stem-and-leaf diagram to represent this set of data. [1]

Answer: ii) Above 945 iii) Doesn't allow all data items **61/N13/4**

---

19 The weights, x kilograms, of 144 people were recorded. The results are summarised in the cumulative frequency table below.

Weight (x kilograms)	$x < 40$	$x < 50$	$x < 60$	$x < 65$	$x < 70$	$x < 90$
Cumulative frequency	0	12	34	64	92	144

(i) On graph paper, draw a cumulative frequency graph to represent these results. [2]

(ii) 64 people weigh more than c kg. Use your graph to find the value of c. [2]

(iii) Calculate estimates of the mean and standard deviation of the weights. [6]

Answer: ii) 67.2 iii) 67.2, 11.3 **63/J13/6**

---

20 A summary of 30 values of x gave the following information:

$$\Sigma(x - c) = 234, \qquad \Sigma(x - c)^2 = 1957.5,$$

where c is a constant.

(i) Find the standard deviation of these values of x. [2]

(ii) Given that the mean of these values is 86, find the value of c. [2]

Answer: i) 2.1 ii) 78.2 **61/J13/1**

21 The following back-to-back stem-and-leaf diagram shows the annual salaries of a group of 39 females and 39 males.

	Females		Males	
(4)	5 2 0 0	20	3	(1)
(9)	9 8 8 7 6 4 0 0 0	21	0 0 7	(3)
(8)	8 7 5 3 3 1 0 0	22	0 0 4 5 6 6	(6)
(6)	6 4 2 1 0 0	23	0 0 2 3 3 5 6 7 7	(9)
(6)	7 5 4 0 0 0	24	0 1 1 2 5 5 6 8 8 9	(10)
(4)	9 5 0 0	25	3 4 5 7 7 8 9	(7)
(2)	5 0	26	0 4 6	(3)

Key: 2 | 20 | 3 means \$20 200 for females and \$20 300 for males.

(i) Find the median and the quartiles of the females' salaries. [2]

You are given that the median salary of the males is \$24 000, the lower quartile is \$22 600 and the upper quartile is \$25 300.

(ii) Represent the data by means of a pair of box-and-whisker plots in a single diagram on graph paper. [3]

Answer: i) M=22700, LQ=21700 UQ= 240000 **61/J13/3**

---

22 In a survey, the percentage of meat in a certain type of take-away meal was found. The results, to the nearest integer, for 193 take-away meals are summarised in the table.

Percentage of meat	1 – 5	6 – 10	11 – 20	21 – 30	31 – 50
Frequency	59	67	38	18	11

(i) Calculate estimates of the mean and standard deviation of the percentage of meat in these take-away meals. [4]

(ii) Draw, on graph paper, a histogram to illustrate the information in the table. [5]

Answer: i) 11.4 **63/N12/4**

---

23 The amounts of money, x dollars, that 24 people had in their pockets are summarised by $\Sigma(x-36) = -60$ and $\Sigma(x-36)^2 = 227.76$. Find Σx and Σx^2. [5]

Answer: 804, 27011.76 **61/N12/2**

24 Prices in dollars of 11 caravans in a showroom are as follows.

16 800 18 500 17 700 14 300 15 500 15 300 16 100 16 800 17 300 15 400 16 400

(i) Represent these prices by a stem-and-leaf diagram. [3]

(ii) Write down the lower quartile of the prices of the caravans in the showroom. [1]

(iii) 3 different caravans in the showroom are chosen at random and their prices are noted. Find the probability that 2 of these prices are more than the median and 1 is less than the lower quartile. [3]

Answer: ii) 15400 iii) 0.121 **61/N12/4**

---

25 Ashfaq and Kuljit have done a school statistics project on the prices of a particular model of headphones for MP3 players. Ashfaq collected prices from 21 shops. Kuljit used the internet to collect prices from 163 websites.

(i) Name a suitable statistical diagram for Ashfaq to represent his data, together with a reason for choosing this particular diagram. [2]

(ii) Name a suitable statistical diagram for Kuljit to represent her data, together with a reason for choosing this particular diagram. [2]

Answer: i) Stem & Leaf ii) Hist. Box & Whisk, CF **63/J12/1**

---

26 The heights, x cm, of a group of young children are summarised by

$$\Sigma(x - 100) = 72, \qquad \Sigma(x - 100)^2 = 499.2.$$

The mean height is 104.8 cm.

(i) Find the number of children in the group. [2]

(ii) Find $\Sigma(x - 104.8)^2$. [3]

Answer: i) 15 ii) 153.6 **63/J12/2**

27 The lengths of the diagonals in metres of the 9 most popular flat screen TVs and the 9 most popular conventional TVs are shown below.

Flat screen :	0.85	0.94	0.91	0.96	1.04	0.89	1.07	0.92	0.76
Conventional :	0.69	0.65	0.85	0.77	0.74	0.67	0.71	0.86	0.75

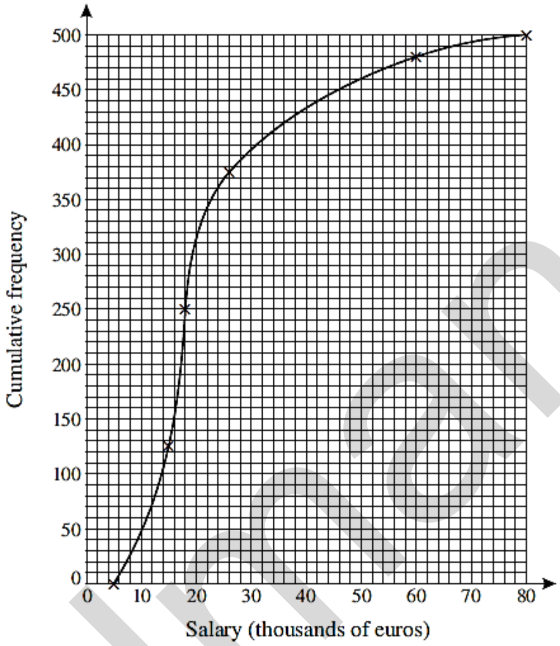
(i) Represent this information on a back-to-back stem-and-leaf diagram. [4]

(ii) Find the median and the interquartile range of the lengths of the diagonals of the 9 conventional TVs. [3]

(iii) Find the mean and standard deviation of the lengths of the diagonals of the 9 flat screen TVs. [2]

Answer: ii) 0.74, 0.13 iii) 0.927, .0882 **61/J12/5**

28



The cumulative frequency graph shows the annual salaries, in thousands of euros, of a random sample of 500 adults with jobs, in France. It has been plotted using grouped data. You may assume that the lowest salary is 5000 euros and the highest salary is 80 000 euros.

(i) On graph paper, draw a box-and-whisker plot to illustrate these salaries. [4]

(ii) Comment on the salaries of the people in this sample. [1]

(iii) An 'outlier' is defined as any data value which is more than 1.5 times the interquartile range above the upper quartile, or more than 1.5 times the interquartile range below the lower quartile.

(a) How high must a salary be in order to be classified as an outlier? [3]

(b) Show that none of the salaries is low enough to be classified as an outlier. [1]

Answer: iii a) 42500 b) -1.5 lower outlier **63/N11/5**

---

29 The values, x, in a particular set of data are summarised by

$$\Sigma(x - 25) = 133, \qquad \Sigma(x - 25)^2 = 3762.$$

The mean, $\bar{x}$, is 28.325.

(i) Find the standard deviation of x.

(ii) Find Σx^2.

Answer: i) 9.11 ii) 35412 **61/N11/2**

---

30 The marks of the pupils in a certain class in a History examination are as follows.

28 33 55 38 42 39 27 48 51 37 57 49 33

The marks of the pupils in a Physics examination are summarised as follows.

Lower quartile: 28, Median: 39, Upper quartile: 67.
The lowest mark was 17 and the highest mark was 74.

(i) Draw box-and-whisker plots in a single diagram on graph paper to illustrate the marks f and Physics.

(ii) State one difference, which can be seen from the diagram, between the marks for H Physics.

Answer: ii) History more spread than physics **61/N11/4**

31 Red Street Garage has 9 used cars for sale. Fairwheel Garage has 15 used cars for sale. The mean age of the cars in Red Street Garage is 3.6 years and the standard deviation is 1.925 years. In Fairwheel Garage, $\Sigma x = 64$ and $\Sigma x^2 = 352$, where x is the age of a car in years.

(i) Find the mean age of all 24 cars. [2]

(ii) Find the standard deviation of the ages of all 24 cars. [4]

Answer: i) 4.02 ii) 2.19 **63/J11/1**

---

32 The following cumulative frequency table shows the examination marks for 300 candidates in country A and 300 candidates in country B.

Mark	<10	<20	<35	<50	<70	<100
Cumulative frequency, A	25	68	159	234	260	300
Cumulative frequency, B	10	46	72	144	198	300

(i) Without drawing a graph, show that the median for country B is higher than the median for country A. [2]

(ii) Find the number of candidates in country A who scored between 20 and 34 marks inclusive. [1]

(iii) Calculate an estimate of the mean mark for candidates in country A. [4]

Answer: ii) 91 iii) 37.6 **63/J11/3**

---

33 There are 5000 schools in a certain country. The cumulative frequency table shows the number of pupils in a school and the corresponding number of schools.

Number of pupils in a school	⩽100	⩽150	⩽200	⩽250	⩽350	⩽450	⩽600
Cumulative frequency	200	800	1600	2100	4100	4700	5000

(i) Draw a cumulative frequency graph with a scale of 2 cm to 100 pupils on the horizontal axis and a scale of 2 cm to 1000 schools on the vertical axis. Use your graph to estimate the median number of pupils in a school. [3]

(ii) 80% of the schools have more than n pupils. Estimate the value of n correct to the nearest ten. [2]

(iii) Find how many schools have between 201 and 250 (inclusive) pupils. [1]

(iv) Calculate an estimate of the mean number of pupils per school. [4]

Answer: i) Median = 270 ii) 160 iii) 500 iv) 268 **61/J11/6**

---

34 Delip measured the speeds, x km per hour, of 70 cars on a road where the speed limit is 60 km per hour. His results are summarised by $\Sigma(x - 60) = 245$.

(i) Calculate the mean speed of these 70 cars. [2]

His friend Sachim used values of $(x - 50)$ to calculate the mean.

(ii) Find $\Sigma(x - 50)$. [2]

(iii) The standard deviation of the speeds is 10.6 km per hour. Calculate $\Sigma(x - 50)^2$. [2]

Answer: i) 63.5 ii) 945 iii) 20623 **63/N10/4**

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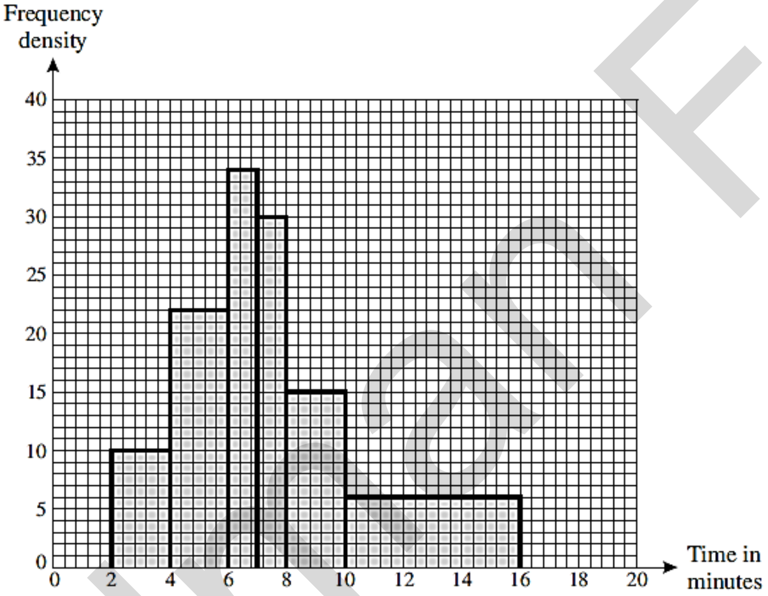
35 The following histogram illustrates the distribution of times, in minutes, that some students spent taking a shower.



(i) Copy and complete the following frequency table for the data. [3]

Time (t minutes)	$2 < t \leqslant 4$	$4 < t \leqslant 6$	$6 < t \leqslant 7$	$7 < t \leqslant 8$	$8 < t \leqslant 10$	$10 < t \leqslant 16$
Frequency						

(ii) Calculate an estimate of the mean time to take a shower. [2]

(iii) Two of these students are chosen at random. Find the probability that exactly one takes between 7 and 10 minutes to take a shower. [3]

63/N10/5

Answer: i)

2 to 4	4 to 6	6 to 7	7 to 8	8 to 10	10 to 16
20	44	34	30	30	36

ii) 7.55 iii) 0.429

---

36 Anita made observations of the maximum temperature, t °C, on 50 days. Her results are summarised by $\Sigma t = 910$ and $\Sigma(t - \bar{t})^2 = 876$, where $\bar{t}$ denotes the mean of the 50 observations. Calculate $\bar{t}$ and the standard deviation of the observations. [3]

Answer: 18.2 , 4.19 **61/N10/1**

37 The weights in grams of a number of stones, measured correct to the nearest gram, are represented in the following table.

Weight (grams)	1 – 10	11 – 20	21 – 25	26 – 30	31 – 50	51 – 70
Frequency	$2x$	$4x$	$3x$	$5x$	$4x$	x

A histogram is drawn with a scale of 1 cm to 1 unit on the vertical axis, which represents frequency density. The 1 – 10 rectangle has height 3 cm.

(i) Calculate the value of x and the height of the 51 – 70 rectangle. [4]

(ii) Calculate an estimate of the mean weight of the stones. [3]

Answer: i) 0.75 ii) 26.6 **61/N10/4**

38 The heights, x cm, of a group of 82 children are summarised as follows.

$$\Sigma(x - 130) = -287, \qquad \text{standard deviation of } x = 6.9.$$

(i) Find the mean height. [2]

(ii) Find $\Sigma(x - 130)^2$. [2]

Answer: i) 126.5 ii) 4908.5 **63/J10/2**

39 The lengths of some insects of the same type from two countries, X and Y, were measured. The stem-and-leaf diagram shows the results.

	Country X		Country Y	
(10)	9 7 6 6 6 4 4 4 3 2	80		
(18)	8 8 8 7 7 6 6 5 5 5 4 4 3 3 3 2 2 0	81	1 1 2 2 3 3 3 5 5 6 7 8 9	(13)
(16)	9 9 9 8 8 7 7 6 5 5 3 2 2 1 0 0	82	0 0 1 2 3 3 3 q 4 5 6 6 7 8 8	(15)
(16)	8 7 6 5 5 5 3 3 2 2 2 1 1 1 0 0	83	0 1 2 2 4 4 4 4 5 5 6 6 7 7 7 8 9	(17)
(11)	8 7 6 5 5 4 4 3 3 1 1	84	0 0 1 2 4 4 5 5 6 6 7 7 7 8 9	(15)
		85	1 2 r 3 3 5 5 6 6 7 8 8	(12)
		86	0 1 2 2 3 5 5 5 8 9 9	(11)

Key: 5 | 81 | 3 means an insect from country X has length 0.815 cm
and an insect from country Y has length 0.813 cm.

(i) Find the median and interquartile range of the lengths of the insects from country X. [2]

(ii) The interquartile range of the lengths of the insects from country Y is 0.028 cm. Find the values of q and r. [2]

(iii) Represent the data by means of a pair of box-and-whisker plots in a single diagram on graph paper. [4]

(iv) Compare the lengths of the insects from the two countries. [2]

Answer: i) 0.825 0.019 ii) 0.824, 0.852 **63/J10/6**

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40 The numbers of people travelling on a certain bus at different times of the day are as follows.

17	5	2	23	16	31	8
22	14	25	35	17	27	12
6	23	19	21	23	8	26

(i) Draw a stem-and-leaf diagram to illustrate the information given above. [3]

(ii) Find the median, the lower quartile, the upper quartile and the interquartile range. [3]

(iii) State, in this case, which of the median and mode is preferable as a measure of central tendency, and why. [1]

Answer: ii) 19, 10, 24, 14 iii) median because mode could be any number occuring twice **61/J10/2**

41 The numbers of rides taken by two students, Fei and Graeme, at a fairground are shown in the following table.

	Roller coaster	Water slide	Revolving drum
Fei	4	2	0
Graeme	1	3	6

(i) The mean cost of Fei's rides is \$2.50 and the standard deviation of the costs of Fei's rides is \$0. Explain how you can tell that the roller coaster and the water slide each cost \$2.50 per ride. [2]

(ii) The mean cost of Graeme's rides is \$3.76. Find the standard deviation of the costs of Graeme's rides. [5]

Answer: i) s.d=0 ii) 1.03 **61/J10/4**

---

42 The amounts spent by 160 shoppers at a supermarket are summarised in the following table.

Amount spent (\$$x$)	$0 < x \leqslant 30$	$30 < x \leqslant 50$	$50 < x \leqslant 70$	$70 < x \leqslant 90$	$90 < x \leqslant 140$
Number of shoppers	16	40	48	26	30

(i) Draw a cumulative frequency graph of this distribution. [4]

(ii) Estimate the median and the interquartile range of the amount spent. [3]

(iii) Estimate the number of shoppers who spent more than \$115. [2]

(iv) Calculate an estimate of the mean amount spent. [2]

Answers: **(ii)** 60, 40 **(iii)** 9 **(iv)** 64.1 **J16/61/Q7**

43 A group of children played a computer game which measured their time in seconds to perform a certain task. A summary of the times taken by girls and boys in the group is shown below.

	Minimum	Lower quartile	Median	Upper quartile	Maximum
Girls	5	5.5	7	9	13
Boys	4	6	8.5	11	16

(i) On graph paper, draw two box-and-whisker plots in a single diagram to illustrate the times taken by girls and boys to perform this task. [3]

(ii) State two comparisons of the times taken by girls and boys. [2]

The strongest candidates produced two statements comparing girls and boys from two of the categories Location, Spread and Distribution and used statistical vocabulary e.g. range or interquartile range and median. If they mentioned skewness they explained that it was positive skew.

J16/63/Q2

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44 The monthly rental prices, $\$x$, for 9 apartments in a certain city are listed and are summarised as follows.

$$\Sigma(x - c) = 1845 \qquad \Sigma(x - c)^2 = 477\,450$$

The mean monthly rental price is \$2205.

(i) Find the value of the constant c. [2]

(ii) Find the variance of these values of x. [2]

(iii) Another apartment is added to the list. The mean monthly rental price is now \$2120.50. Find the rental price of this additional apartment. [2]

Answers: (i) 2 000 (ii) 11 025 (iii) 1 360

J16/63/Q4

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45 Kadijat noted the weights, x grams, of 30 chocolate buns. Her results are summarised by

$$\Sigma(x - k) = 315, \qquad \Sigma(x - k)^2 = 4022,$$

where k is a constant. The mean weight of the buns is 50.5 grams.

(i) Find the value of k. [2]

(ii) Find the standard deviation of x. [2]

Answer: 40, 4.88

J17/61/Q1

46 The times taken, t seconds, by 1140 people to solve a puzzle are summarised in the table.

Time (t seconds)	$0 \leqslant t < 20$	$20 \leqslant t < 40$	$40 \leqslant t < 60$	$60 \leqslant t < 100$	$100 \leqslant t < 140$
Number of people	320	280	220	220	100

(i) On the grid, draw a histogram to illustrate this information. [4]

(ii) Calculate an estimate of the mean of t. [2]

Answer: (ii) 45.8

J17/61/Q4

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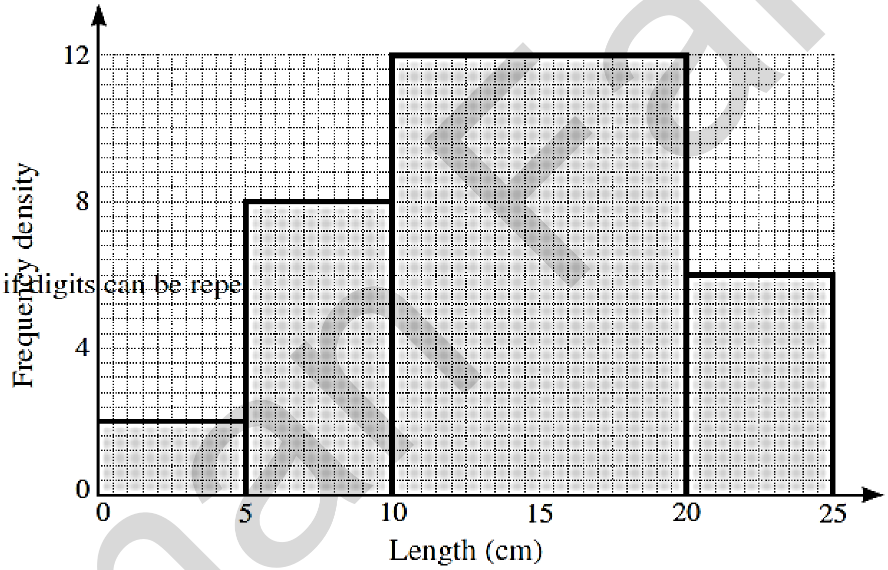
47 The following histogram represents the lengths of worms in a garden.



(ii) if digits can be repe [3

(i) Calculate the frequencies represented by each of the four histogram columns. [2]

(ii) On the grid on the next page, draw a cumulative frequency graph to represent the lengths of worms in the garden. [4]

(iii) Use your graph to estimate the median and interquartile range of the lengths of worms in the garden. [3]

(iv) Calculate an estimate of the mean length of worms in the garden. [2]

Answers: (i) 10, 40, 120, 30 (iii) median = 14.2; IQ range = 8.5 (iv) 14

J17/63/Q7

48 The masses, in grams, of components made in factory A and components made in factory B are shown below.

Factory A	0.049	0.050	0.053	0.054	0.057	0.058	0.058
	0.059	0.061	0.061	0.061	0.063	0.065	
Factory B	0.031	0.056	0.049	0.044	0.038	0.048	0.051
	0.064	0.035	0.042	0.047	0.054	0.058	

(i) Draw a back-to-back stem-and-leaf diagram to represent the masses of components made in the two factories. [5]

(ii) Find the median and the interquartile range for the masses of components made in factory B. [3]

(iii) Make two comparisons between the masses of components made in factory A and the masses of those made in factory B. [2]

Answers (ii) median 0.048g IQR 0.015g **N16/61/Q7**

---

49 The tables summarise the heights, h cm, of 60 girls and 60 boys.

Height of girls (cm)	$140 < h \leqslant 150$	$150 < h \leqslant 160$	$160 < h \leqslant 170$	$170 < h \leqslant 180$	$180 < h \leqslant 190$
Frequency	12	21	17	10	0

Height of boys (cm)	$140 < h \leqslant 150$	$150 < h \leqslant 160$	$160 < h \leqslant 170$	$170 < h \leqslant 180$	$180 < h \leqslant 190$
Frequency	0	20	23	12	5

(i) On graph paper, using the same set of axes, draw two cumulative frequency graphs to illustrate the data. [4]

(ii) On a school trip the students have to enter a cave which is 165 cm high. Use your graph to estimate the percentage of the girls who will be unable to stand upright. [3]

(iii) The students are asked to compare the heights of the girls and the boys. State one advantage of using a pair of box-and-whisker plots instead of the cumulative frequency graphs to do this. [1]

Answers: (ii) 30% **N16/63/Q5**

50 The time taken by a car to accelerate from 0 to 30 metres per second was measured correct to the nearest second. The results from 48 cars are summarised in the following table.

Time (seconds)	3 – 5	6 – 8	9 – 11	12 – 16	17 – 25
Frequency	10	15	17	4	2

(i) On the grid, draw a cumulative frequency graph to represent this information. [3]

(ii) 35 of these cars accelerated from 0 to 30 metres per second in a time more than t seconds. Estimate the value of t. [2]

Answer: (ii) 6.5 sec

N17/61/Q2

---

51 The ages of a group of 12 people at an Art class have mean 48.7 years and standard deviation 7.65 years. The ages of a group of 7 people at another Art class have mean 38.1 years and standard deviation 4.2 years.

(i) Find the mean age of all 19 people. [2]

(ii) The individual ages in years of people in the first Art class are denoted by x and those in the second Art class by y. By first finding Σx^2 and Σy^2, find the standard deviation of the ages of all 19 people. [4]

N17/61/Q4

Answers: (i) 44.8 (ii) 8.36 or 8.37

---

52 Tien measured the arm lengths, x cm, of 20 people in his class. He found that $\Sigma x = 1218$ and the standard deviation of x was 4.2. Calculate $\Sigma(x - 45)$ and $\Sigma(x - 45)^2$. [3]

Answers: 318, 5409

N17/63/Q2

53 The number of Olympic medals won in the 2012 Olympic Games by the top 27 countries is shown below.

104	88	82	65	44	38	35	34	28
28	18	18	17	17	14	13	13	12
12	10	10	10	9	6	5	2	2

(i) Draw a stem-and-leaf diagram to illustrate the data. [4]

(ii) Find the median and quartiles and draw a box-and-whisker plot on the grid. [5]

Answers: (ii) Median = 17, LQ =10, UQ = 35

N17/63/Q5

---

54 The daily rainfall, x mm, in a certain village is recorded on 250 consecutive days. The results are summarised in the following cumulative frequency table.

Rainfall, x mm	$x \leqslant 20$	$x \leqslant 30$	$x \leqslant 40$	$x \leqslant 50$	$x \leqslant 70$	$x \leqslant 100$
Cumulative frequency	52	94	142	172	222	250

(i) On the grid, draw a cumulative frequency graph to illustrate the data. [2]

(ii) On 100 of the days, the rainfall was k mm or more. Use your graph to estimate the value of k. [2]

(iii) Calculate estimates of the mean and standard deviation of the daily rainfall in this village. [6]

Answers: (ii) 42 (iii) $\mu = 39.9$ $\sigma = 23.2$

N18/61/Q6

---

55 The heights, in cm, of the 11 members of the Anvils athletics team and the 11 members of the Brecons swimming team are shown below.

Anvils	173	158	180	196	175	165	170	169	181	184	172
Brecons	166	170	171	172	172	178	181	182	183	183	192

(i) Draw a back-to-back stem-and-leaf diagram to represent this information, with Anvils on the left-hand side of the diagram and Brecons on the right-hand side. [4]

(ii) Find the median and the interquartile range for the heights of the Anvils. [3]

The heights of the 11 members of the Anvils are denoted by x cm. It is given that $\Sigma x = 1923$ and $\Sigma x^2 = 337\,221$. The Anvils are joined by 3 new members whose heights are 166 cm, 172 cm and 182 cm.

(iii) Find the standard deviation of the heights of all 14 members of the Anvils. [4]

N18/63/Q7

Anvils		Brecons
8	15	
9 5	16	6
5 3 2 0	17	0 1 2 2 8
4 1 0	18	1 2 3 3
6	19	2

Key: 5|16|6 means 165 cm for Anvils and 166 cm for Brecons

Answer: median = 173; IQ range = 12

Answer: 9.19

PROBABILITY

> The relative frequency of an event happening can be used as an **estimate** of the probability of that event happening. The estimate is more likely to be close to the true probability if the experiment has been carried out a large number of times.

THE COMPLEMENTARY EVENT A'

It is always worth watching for this short cut, and it is also useful to have some language to describe it. If A is an event, the event 'not A' is the event consisting of those outcomes in the sample space which are not in A. Since the sum of the probabilities assigned to outcomes in the sample space is 1,

$$P(A) + P(\text{not } A) = 1.$$

The event 'not A' is called the **complement** of the event A. The symbol A' is used to denote the complement of A.

> If A is an event, then A' is the complement of A, and
>
> $$P(A) + P(A') = 1. \qquad (4.1)$$

Example 2.1

It was found that, out of a box of 50 matches, 45 lit but the others did not.
What was the probability that a randomly selected match would not have lit?

An **event** is a set of possible outcomes from an experiment.

So for rolling a die, you could say:

A is the event that a five is rolled.

B is the event that an even number is rolled.

C is the event that an odd number is rolled.

$A \cup B$ is the **union** of events *A* and *B*.
This means *A* or *B* or both can happen.
Rolling 2, 4, 5, 6 are the outcomes which satisfy $A \cup B$.

$A \cap C$ is the **intersection** of events *A* and *C*.
This means both *A* and *C* have to happen.
Rolling 5 is the only outcome that satisfies $A \cap C$.

A' means the event 'A does not happen'.

This is the **complementary event**, and $P(A') = 1 - P(A)$.

Rolling 1, 2, 3, 4, 6 are the outcomes which satisfy A'.

Note that $B \cap C$ has no outcomes satisfying it – there are no numbers which are both even and odd. This can be written as $B \cap C = \varnothing$ or $B \cap C = \{\ \}$ and is referred to as the **null set** or **empty set**.

Sometimes the complementary probability is much easier to work out directly.

ADDITION LAW OF PROBABILITIES

If we take all the outcomes that satisfy A, and then all the outcomes that satisfy B, then any outcomes which satisfy both will be double counted, so

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Example 2.2

Events C and D are such that P(C) $= \frac{19}{30}, P(D) = \frac{2}{5}\ and\ P(\ C\ U\ D) = \frac{4}{5}$,

Find P(C ∩ D).

POSSIBILITY DIAGRAMS

Example 2.3

Two dice are thrown. The scores on the dice are added.

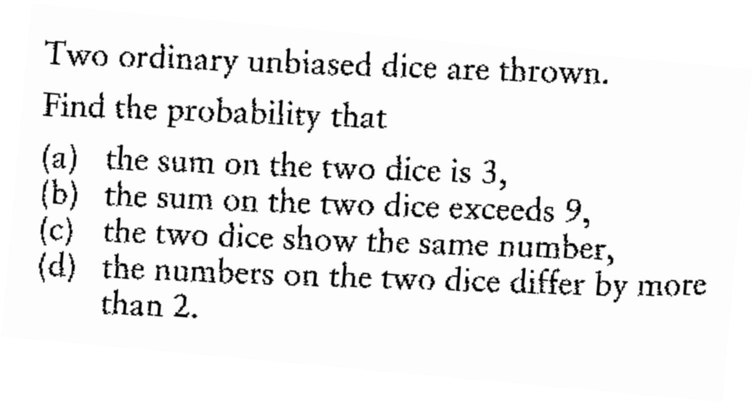
i. Copy and complete this table showing all the possible outcomes.

		First die					
		1	2	3	4	5	6
Second die	1						
	2						
	3						
	4						10
	5						11
	6	7	8	9	10	11	12

ii. What is the probability of a score of 4?

iii. What is the most likely outcome

Example 2.4



Two ordinary unbiased dice are thrown.

Find the probability that

(a) the sum on the two dice is 3,

(b) the sum on the two dice exceeds 9,

(c) the two dice show the same number,

(d) the numbers on the two dice differ by more than 2.

TREE DIAGRAMS

Example 2.5

For a person living in a particular town, the probability that during a period of one year they will start a new job is 0.06. The probability that they will be fired from a job is 0.03.

Assuming these events are independent, draw a tree diagram to represent this information.

Find the probability that during one year a randomly selected person living in the town has

a) neither of these events happen

b) exactly one of these events happen

c) both of these events happen.

Example 2.6

Every Saturday, a man invites his sister to the theatre or to the cinema. 70% of his invitations are to the theatre and 90% of these are accepted. His sister rejects 40% of his invitations to the cinema.

Find the probability that the brother's invitation is accepted on any particular Saturday.

Example 2.7

Ruth drives her car to work – provided she can get it to start! When she remembers to put the car in the garage the night before, it starts next morning with a probability of 0.95. When she forgets to put the car away, it starts next morning with a probability of 0.75. She remembers to put her car in the garage 90% of the time.

What is the probability that Ruth driver her car to work on a randomly chosen day?

Example 2.8

There are four red, three green and five blue discs in a bag. Two discs are drawn out.

Find the probability that two discs the same colour are drawn.

Example 2.9

A bag contains seven black and three white marbles. Three marbles are chosen at random and in succession, each marble being replaced after it has been taken out of the bag.

Draw a tree diagram to show all possible selections.

From your diagram, or otherwise, calculate, to two significant figures, the probability of choosing

(a) three black marbles,
(b) a white marble, a black marble and a white marble in that order,
(c) two white marbles and a black marble in any order,
(d) at least one black marble.

Example 2.10

Abha passes through three independent sets of traffic lights when she drives to work. The probability that she has to stop at any particular set of lights is 0.2. Find the probability that Abha:

a first has to stop at the second set of lights

b has to stop at exactly one set of lights

c has to stop at any set of lights.

EXCLUSIVE (OR MUTUALLY EXCLUSIVE) EVENTS

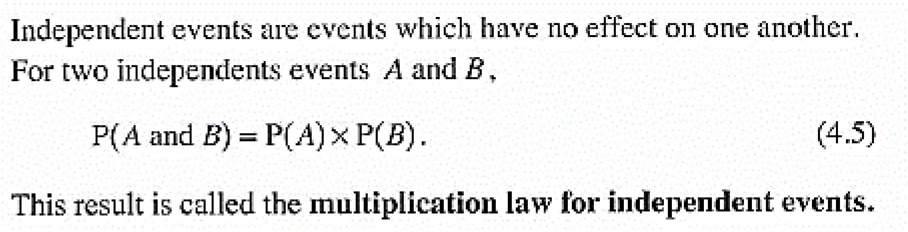
Example 2.11

In a race in which there are no dead heats, the probability that John wins is 0.3, the probability that Paul win is 0.2 and the probability that Mark wins is 0.4.
Find the probability that
(a) John or Mark wins,
(b) John or Paul or Mark wins,
(c) someone else wins.

Example 2.12

A card is drawn from an ordinary pack of 52 playing cards. Find the probability that the card is
(a) a club or a diamond
(b) a club or a King.

INDEPENDENT EVENTS



Independent events are events which have no effect on one another.
For two independents events A and B,

$$P(A \text{ and } B) = P(A) \times P(B). \qquad (4.5)$$

This result is called the **multiplication law for independent events.**

Example 2.13

Events A and B are independent and $P(A)=\frac{1}{3}$, $P(A \cap B) = \frac{1}{12}$
Find (a) P(B) (b) $P(A \cup B)$

Example 2.14

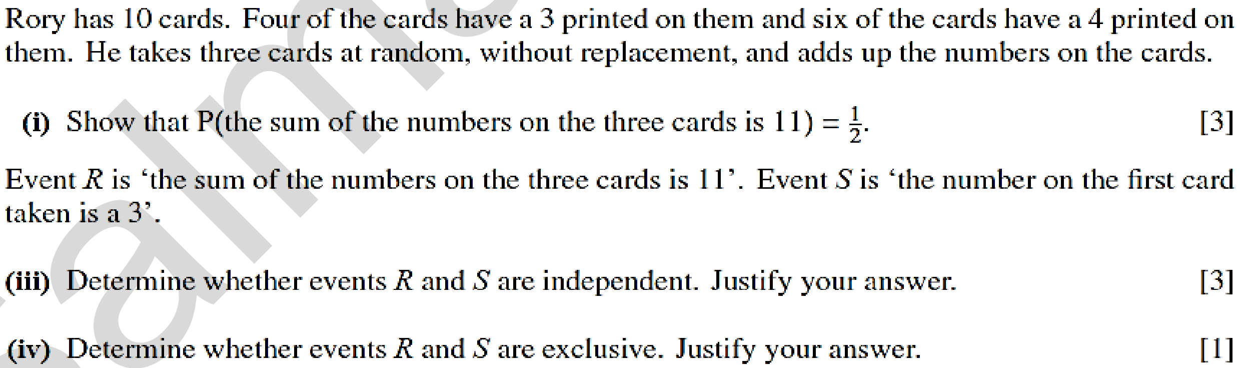
The events A and B are such that
P(A) = 0.45, P(B) = 0.35 and P(A U B) = 0.7
(a) Find the value of $P(A \cap B)$
(b) Explain why the events A and B are not independent,

Example 2.15

Two ordinary fair dice, one red and one blue are to be rolled once.
(a) Find the probabilities of the following events:
Event A: the number showing on the red die will be a 5 or a 6
Event B: the total of the numbers showing on the two dice will be 7,
Event C: the total of the numbers showing the two dice will be 8.
(b) State, with a reason, which two of the events A, B and C are mutually exclusive.
(c) Show that the events A and B are independent.

Example 2.16

N13/62/Q7



Rory has 10 cards. Four of the cards have a 3 printed on them and six of the cards have a 4 printed on them. He takes three cards at random, without replacement, and adds up the numbers on the cards.

(i) Show that P(the sum of the numbers on the three cards is 11) = $\frac{1}{2}$. [3]

Event R is 'the sum of the numbers on the three cards is 11'. Event S is 'the number on the first card taken is a 3'.

(iii) Determine whether events R and S are independent. Justify your answer. [3]

(iv) Determine whether events R and S are exclusive. Justify your answer. [1]

CONDITIONAL PROBABILITY

Example 2.17

A company is worried about the high turnover of its employees and decides to investigate whether they are more likely to stay if they are given training. On 1 January one year the company employed

256 people (excluding those about to retire). During the year a record was kept of who received training as well as who left the company. The results are summarized in this table.

	Still employed	Left company	Total
Given training	109	43	152
Not given training	60	44	104
Totals	169	87	256

i. Find the probability that a randomly selected employee
    a. Received training
    b. Received training and did not leave the company.
ii. Are the events T and S independent?
iii. Find the probabiltiy that a randomly selected employee
    a. Did not leave the company, given that the person had received training
    b. Did not leave the company, given that the person had not received training.

Example 2.18

Data about employment for males and females in a small rural area are shown in the table.

	Unemployed	Employed
Male	206	412
Female	358	305

A person from this area is chosen at random. Let M be the event that the person is male and let E be the event that the person is employed.

(i) Find P(M).

(ii) Find P(M and E).

(iii) Are M and E independent events? Justify your answer.

(iv) Given that the person chosen is unemployed, find the probability that the person is female.

[Cambridge International AS and A Level Mathematics 9709, Paper 6 Q5 June 2005]

Example 2.19

A medical centre encourages elderly people to have a flu vaccination each year. The vaccination reduces the likelihood of getting flu from 40% to 10%.

If 45% of the elderly people visiting the centre have the vaccination, find the probability that an elderly person chosen at random

a) gets flu **b)** had the vaccination, given that they get flu.

Example 2.20

X and Y are two events such that P(X|Y) = 0.4, P(Y) = 0.25 and P(X) = 0.2
Find
(a) P (Y|X)   (b) P(X ∩ Y)   (c) P(X U Y)

Example 2.21

Maria chooses toast for her breakfast with probability 0.85. If she does not choose toast then she has a bread roll. If she chooses toast then the probability that she will have jam on it is 0.8. If she has a bread roll then the probability that she will have jam on it is 0.4.

i Draw a fully labelled tree diagram to show this information. **[2]**

ii Given that Maria did **not** have jam for breakfast, find the probability that she had toast. **[4]**

Cambridge International AS & A Level Mathematics 9709 Paper 62 Q3 November 2009

Example 2.22

When a person needs a minicab, it is hired from one of three firms, X Y and Z. Of the hirings 40% are from, X, 50% are from Y and 10% are from Z. For cabs hired from X, 9% arrive late, the corresponding percentages for cabs hired from firms Y and Z being 6% and 20% respectively. Calculate the probability that the next cab hired

(a) will be from X and will not arrive late,
(b) will arrive late.
(c) Given that a call is made for a minicab and that it arrives late, find, to three decimal places, the probability that it came from Y.

Example 2.23

There are three sets of traffic lights on Karinne's journey to work. The independent probabilities that Karinne has to stop at the first, second and third set of lights are 0.4, 0.8 and 0.3 respectively.

(i) Draw a tree diagram to show this information.
(ii) Find the probability that Karinne has to stop at each of the first two sets of lights but does not have to stop at the third set.
(iii) Find the probability that Karinne has to stop at exactly two of the three sets of lights.
(iv) Find the probability that Karinne has to stop at the first set of lights, given that she has to stop at exactly two sets of lights.

[Cambridge International AS and A Level Mathematics 9709, Paper 6 Q6 November 2008]

Example 2.24

A die is biased so that, when it is rolled, the probability of obtaining a score of 6 is $\frac{1}{4}$. The probabilities of obtaining each of the other five scores 1,2,3,4,5 are all equal.

(a) Calculate the probability of obtaining a score of five with this biased die.

(b) The biased die and an unbiased die are now rolled together. Calculate the probability that the total score is 11 or more.

(c) The two dice are rolled again. Given that the total score is 11 or more, calculate the probability that the score on the biased die is 6.

Example 2.25

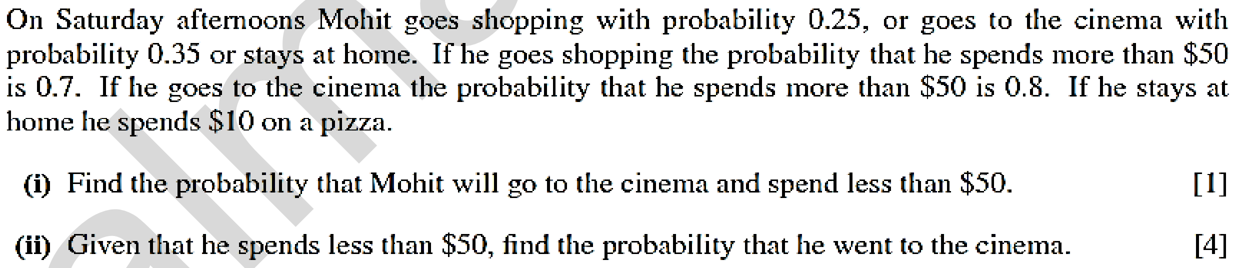
A factory has three machines A, B, C producing large numbers of a certain item. Of the total daily item, 50% are produced on A, 30% on B and 20% on C.

Records show that 2% of item produced on A are defective, 3% of items produced on B are defective and 4% of items produced on C are defective. The occurrence of a defective item is independent of all other items.

One item is chosen at random from a day's total output.
(a) Show that the probability of its being defective 0.027.
(b) Given that it is defective, find the probability that it was produced on machine A.

Example 2.26

N13/62/Q2



On Saturday afternoons Mohit goes shopping with probability 0.25, or goes to the cinema with probability 0.35 or stays at home. If he goes shopping the probability that he spends more than \$50 is 0.7. If he goes to the cinema the probability that he spends more than \$50 is 0.8. If he stays at home he spends \$10 on a pizza.

(i) Find the probability that Mohit will go to the cinema and spend less than \$50. [1]

(ii) Given that he spends less than \$50, find the probability that he went to the cinema. [4]

HOMEWORK:PROBABILITY– VARIANT 62

1 The people living in 3 houses are classified as children (C), parents (P) or grandparents (G). Th numbers living in each house are shown in the table below.

House number 1	House number 2	House number 3
$4C$, $1P$, $2G$	$2C$, $2P$, $3G$	$1C$, $1G$

(i) All the people in all 3 houses meet for a party. One person at the party is chosen at random Calculate the probability of choosing a grandparent. [2

(ii) A house is chosen at random. Then a person in that house is chosen at random. Using a tre diagram, or otherwise, calculate the probability that the person chosen is a grandparent. [3

(iii) Given that the person chosen by the method in part **(ii)** is a grandparent, calculate the probabilit that there is also a parent living in the house. [4

Answers: (i) $\frac{3}{8}$; (ii) $\frac{17}{42}$; (iii) $\frac{10}{17}$.

J03/Q6

---

2 Data about employment for males and females in a small rural area are shown in the table.

	Unemployed	Employed
Male	206	412
Female	358	305

A person from this area is chosen at random. Let M be the event that the person is male and let E be the event that the person is employed.

(i) Find P(M). [2]

(ii) Find P(M and E). [1]

(iii) Are M and E independent events? Justify your answer. [3]

(iv) Given that the person chosen is unemployed, find the probability that the person is female. [2]

Answers: (i) $\frac{618}{128}$; (ii) $\frac{412}{1281}$; (iv) $\frac{358}{564}$.

J05/Q5

3 When Don plays tennis, 65% of his first serves go into the correct area of the court. If the first serve goes into the correct area, his chance of winning the point is 90%. If his first serve does not go into the correct area, Don is allowed a second serve, and of these, 80% go into the correct area. If the second serve goes into the correct area, his chance of winning the point is 60%. If neither serve goes into the correct area, Don loses the point.

(i) Draw a tree diagram to represent this information. [4]

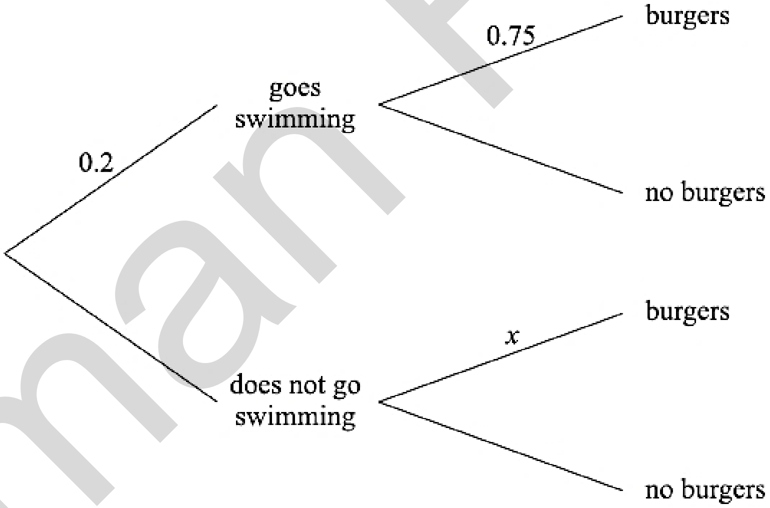
(ii) Using your tree diagram, find the probability that Don loses the point. [3]

(iii) Find the conditional probability that Don's first serve went into the correct area, given that he loses the point. [2]

Answers: (ii) 0.247; (iii) $\frac{5}{19}$ = 0.263. **J04/Q6**

---

4 The probability that Henk goes swimming on any day is 0.2. On a day when he goes swimming, the probability that Henk has burgers for supper is 0.75. On a day when he does not go swimming the probability that he has burgers for supper is x. This information is shown on the following tree diagram.



The probability that Henk has burgers for supper on any day is 0.5.

(i) Find x. [4]

(ii) Given that Henk has burgers for supper, find the probability that he went swimming that day. [2]

Answers: (i) 0.4375; (ii) 0.3. **J06/Q2**

5 Jamie is equally likely to attend or not to attend a training session before a football match. If he attends, he is certain to be chosen for the team which plays in the match. If he does not attend, there is a probability of 0.6 that he is chosen for the team.

(i) Find the probability that Jamie is chosen for the team. [3]

(ii) Find the conditional probability that Jamie attended the training session, given that he was chosen for the team. [3]

Answers: (i) 0.8; (ii) 0.625. **J07/Q2**

---

6 In country A 30% of people who drink tea have sugar in it. In country B 65% of people who drink tea have sugar in it. There are 3 million people in country A who drink tea and 12 million people in country B who drink tea. A person is chosen at random from these 15 million people.

(i) Find the probability that the person chosen is from country A. [1]

(ii) Find the probability that the person chosen does not have sugar in their tea. [2]

(iii) Given that the person chosen does not have sugar in their tea, find the probability that the person is from country B. [2]

Answers: (i) 0.2; (ii) 0.42; (iii) $\frac{2}{3}$ or 0.667. **J08/Q2**

---

7 At a zoo, rides are offered on elephants, camels and jungle tractors. Ravi has money for only one ride. To decide which ride to choose, he tosses a fair coin twice. If he gets 2 heads he will go on the elephant ride, if he gets 2 tails he will go on the camel ride and if he gets 1 of each he will go on the jungle tractor ride.

(i) Find the probabilities that he goes on each of the three rides. [2]

The probabilities that Ravi is frightened on each of the rides are as follows:

elephant ride $\frac{6}{10}$, camel ride $\frac{7}{10}$, jungle tractor ride $\frac{8}{10}$.

(ii) Draw a fully labelled tree diagram showing the rides that Ravi could take and whether or not he is frightened. [2]

Ravi goes on a ride.

(iii) Find the probability that he is frightened. [2]

(iv) Given that Ravi is **not** frightened, find the probability that he went on the camel ride. [3]

J09/Q5

Answers: (i) $\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{2}$; (ii) $\frac{29}{40}$; (iii) $\frac{3}{11}$.

8 Two fair twelve-sided dice with sides marked 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 are thrown, and the numbers on the sides which land face down are noted. Events Q and R are defined as follows.

Q : the product of the two numbers is 24.
R : both of the numbers are greater than 8.

(i) Find $\mathrm{P}(Q)$. [2]

(ii) Find $\mathrm{P}(R)$. [2]

(iii) Are events Q and R exclusive? Justify your answer. [2]

(iv) Are events Q and R independent? Justify your answer. [2]

Answers: (i) $\frac{1}{24}$; (ii) $\frac{1}{9}$; (iii) Since $\mathrm{P}(Q \cap R) = 0$, yes, they are mutually exclusive; (iv) Since $\mathrm{P}(Q) \times \mathrm{P}(R) \neq 0$, they are not independent.

J10/62/Q5

---

9 Ivan throws three fair dice.

(i) List all the possible scores on the three dice which give a total score of 5, and hence show that the probability of Ivan obtaining a total score of 5 is $\frac{1}{36}$. [3]

(ii) Find the probability of Ivan obtaining a total score of 7. [3]

Answer: (ii) $\frac{5}{72}$.

N02/Q2

---

10 Rachel and Anna play each other at badminton. Each game results in either a win for Rachel or a win for Anna. The probability of Rachel winning the first game is 0.6. If Rachel wins a particular game, the probability of her winning the next game is 0.7, but if she loses, the probability of her winning the next game is 0.4. By using a tree diagram, or otherwise,

(i) find the conditional probability that Rachel wins the first game, given that she loses the second, [5]

(ii) find the probability that Rachel wins 2 games and loses 1 game out of the first three games they play. [4]

Answers: (i) 0.429; (ii) 0.31.

N02/Q5

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11 Single cards, chosen at random, are given away with bars of chocolate. Each card shows a picture of one of 20 different football players. Richard needs just one picture to complete his collection. He buys 5 bars of chocolate and looks at all the pictures. Find the probability that

(i) Richard does not complete his collection, [2]

(ii) he has the required picture exactly once, [2]

(iii) he completes his collection with the third picture he looks at. [2]

Answers: (i) 0.774; (ii) 0.204; (iii) 0.0451. **N03/Q4**

---

12 When Andrea needs a taxi, she rings one of three taxi companies, A, B or C. 50% of her calls are to taxi company A, 30% to B and 20% to C. A taxi from company A arrives late 4% of the time, a taxi from company B arrives late 6% of the time and a taxi from company C arrives late 17% of the time.

(i) Find the probability that, when Andrea rings for a taxi, it arrives late. [3]

(ii) Given that Andrea's taxi arrives late, find the conditional probability that she rang company B. [3]

N04/Q3

Answers: (i) 0.072; (ii) 0.25.

---

13 Boxes of sweets contain toffees and chocolates. Box A contains 6 toffees and 4 chocolates, box B contains 5 toffees and 3 chocolates, and box C contains 3 toffees and 7 chocolates. One of the boxes is chosen at random and two sweets are taken out, one after the other, and eaten.

(i) Find the probability that they are both toffees. [3]

(ii) Given that they are both toffees, find the probability that they both came from box A. [3]

Answers: (i) 0.252; (ii) 0.440. **N05/Q2**

---

14 Two fair dice are thrown.

(i) Event A is 'the scores differ by 3 or more'. Find the probability of event A. [3]

(ii) Event B is 'the product of the scores is greater than 8'. Find the probability of event B. [2]

(iii) State with a reason whether events A and B are mutually exclusive. [2]

Answers: (i) $\frac{1}{3}$; (ii) $\frac{5}{9}$. **N06/Q4**

---

15 There are three sets of traffic lights on Karinne's journey to work. The independent probabilities that Karinne has to stop at the first, second and third set of lights are 0.4, 0.8 and 0.3 respectively.

(i) Draw a tree diagram to show this information. [2]

(ii) Find the probability that Karinne has to stop at each of the first two sets of lights but does not have to stop at the third set. [2]

(iii) Find the probability that Karinne has to stop at exactly two of the three sets of lights. [3]

(iv) Find the probability that Karinne has to stop at the first set of lights, given that she has to stop at exactly two sets of lights. [3]

Answers: (ii) 0.224; (iii) 0.392; (iv) 0.633. **N08/Q6**

16 Maria chooses toast for her breakfast with probability 0.85. If she does not choose toast then she has a bread roll. If she chooses toast then the probability that she will have jam on it is 0.8. If she has a bread roll then the probability that she will have jam on it is 0.4.

(i) Draw a fully labelled tree diagram to show this information. [2]

(ii) Given that Maria did **not** have jam for breakfast, find the probability that she had toast. [4]

Answer: (ii) $\frac{17}{26}$ or 0.654.

N09/62/Q3

---

17 A fair five-sided spinner has sides numbered 1, 2, 3, 4, 5. Raj spins the spinner and throws two fair dice. He calculates his score as follows.

- If the spinner lands on an **even-numbered** side, Raj **multiplies** the two numbers showing on the dice to get his score.
- If the spinner lands on an **odd-numbered** side, Raj **adds** the numbers showing on the dice to get his score.

Given that Raj's score is 12, find the probability that the spinner landed on an even-numbered side. [6]

Answer: $\frac{8}{11}$.

N10/62/Q3

---

18 In a certain country 54% of the population is male. It is known that 5% of the males are colour-blind and 2% of the females are colour-blind. A person is chosen at random and found to be colour-blind. By drawing a tree diagram, or otherwise, find the probability that this person is male. [6]

Answer: 0.746.

N03/Q5

19 A box of biscuits contains 30 biscuits, some of which are wrapped in gold foil and some of which are unwrapped. Some of the biscuits are chocolate-covered. 12 biscuits are wrapped in gold foil, and of these biscuits, 7 are chocolate-covered. There are 17 chocolate-covered biscuits in total.

(i) Copy and complete the table below to show the number of biscuits in each category. [2]

	Wrapped in gold foil	Unwrapped	Total
Chocolate-covered			
Not chocolate-covered			
Total			30

A biscuit is selected at random from the box.

(ii) Find the probability that the biscuit is wrapped in gold foil. [1]

The biscuit is returned to the box. An unwrapped biscuit is then selected at random from the box.

(iii) Find the probability that the biscuit is chocolate-covered. [1]

The biscuit is returned to the box. A biscuit is then selected at random from the box.

(iv) Find the probability that the biscuit is unwrapped, given that it is chocolate-covered. [1]

The biscuit is returned to the box. Nasir then takes 4 biscuits without replacement from the box.

(v) Find the probability that he takes exactly 2 wrapped biscuits. [4]

Answers: (ii) 2/5 (iii) 5/9 (iv) 10/17 (v) 0.368 **J12/62/Q6**

---

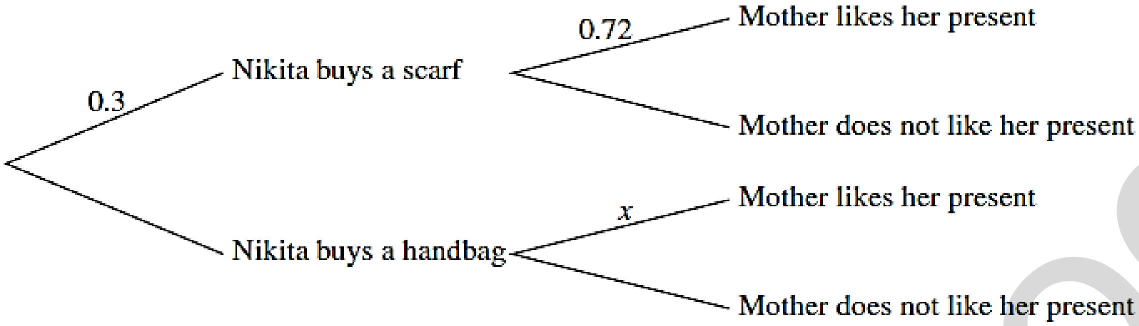
20 Roger and Andy play a tennis match in which the first person to win two sets wins the match. The probability that Roger wins the first set is 0.6. For sets after the first, the probability that Roger wins the set is 0.7 if he won the previous set, and is 0.25 if he lost the previous set. No set is drawn.

(i) Find the probability that there is a winner of the match after exactly two sets. [3]

(ii) Find the probability that Andy wins the match given that there is a winner of the match after exactly two sets. [2]

Answers: 0.72, 0.417 **J14/62/Q3**

21



Nikita goes shopping to buy a birthday present for her mother. She buys either a scarf, with probability 0.3, or a handbag. The probability that her mother will like the choice of scarf is 0.72. The probability that her mother will like the choice of handbag is x. This information is shown on the tree diagram. The probability that Nikita's mother likes the present that Nikita buys is 0.783.

(i) Find x. [3]

(ii) Given that Nikita's mother does not like her present, find the probability that the present is a scarf. [4]

Answers: (i) 0.81, (ii) 0.387 **J15/62/Q4**

---

22 Fabio drinks coffee each morning. He chooses Americano, Cappucino or Latte with probabilities 0.5, 0.3 and 0.2 respectively. If he chooses Americano he either drinks it immediately with probability 0.8, or leaves it to drink later. If he chooses Cappucino he either drinks it immediately with probability 0.6, or leaves it to drink later. If he chooses Latte he either drinks it immediately with probability 0.1, or leaves it to drink later.

(i) Find the probability that Fabio chooses Americano and leaves it to drink later. [1]

(ii) Fabio drinks his coffee immediately. Find the probability that he chose Latte. [4]

Answers: (i) 0.1; (ii) 1/30. **N12/62/Q1**

---

23 On Saturday afternoons Mohit goes shopping with probability 0.25, or goes to the cinema with probability 0.35 or stays at home. If he goes shopping the probability that he spends more than \$50 is 0.7. If he goes to the cinema the probability that he spends more than \$50 is 0.8. If he stays at home he spends \$10 on a pizza.

(i) Find the probability that Mohit will go to the cinema and spend less than \$50. [1]

(ii) Given that he spends less than \$50, find the probability that he went to the cinema. [4]

Answers: 0.07, 0.128 **N13/62/Q2**

24 Sharik attempts a multiple choice revision question on-line. There are 3 suggested answers, one of which is correct. When Sharik chooses an answer the computer indicates whether the answer is right or wrong. Sharik first chooses one of the three suggested answers at random. If this answer is wrong he has a second try, choosing an answer at random from the remaining 2. If this answer is also wrong Sharik then chooses the remaining answer, which must be correct.

(i) Draw a fully labelled tree diagram to illustrate the various choices that Sharik can make until the computer indicates that he has answered the question correctly. [4]

(ii) The random variable X is the number of attempts that Sharik makes up to and including the one that the computer indicates is correct. Draw up the probability distribution table for X and find $\mathrm{E}(X)$. [4]

Answers: **(ii)** $\mathrm{P}(1) = \mathrm{P}(2) = \mathrm{P}(3) = \frac{1}{3}$, $\mathrm{E}(X) = 2$

N14/62/Q4

---

25 One plastic robot is given away free inside each packet of a certain brand of biscuits. There are four colours of plastic robot (red, yellow, blue and green) and each colour is equally likely to occur. Nick buys some packets of these biscuits. Find the probability that

(i) he gets a green robot on opening his first packet, [1]

(ii) he gets his first green robot on opening his fifth packet. [2]

Nick's friend Amos is also collecting robots.

(iii) Find the probability that the first four packets Amos opens all contain different coloured robots. [3]

Answers: **(i)** ¼, **(ii)** 81/1084, **(iii)** 3/32

N15/62/Q3

HOMEWORK:PROBABILITY VARIANT 61 AND 63

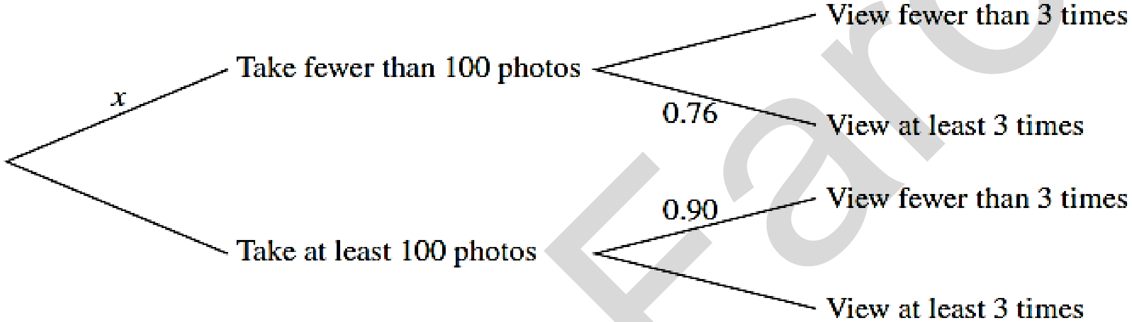
1 Jason throws two fair dice, each with faces numbered 1 to 6. Event A is 'one of the numbers obtained is divisible by 3 and the other number is not divisible by 3'. Event B is 'the product of the two numbers obtained is even'.

(i) Determine whether events A and B are independent, showing your working. [5]

(ii) Are events A and B mutually exclusive? Justify your answer. [1]

Answer: i) Independent ii) mutually exclusive **61/J15/Q3**

2



A survey is undertaken to investigate how many photos people take on a one-week holiday and also how many times they view past photos. For a randomly chosen person, the probability of taking fewer than 100 photos is x. The probability that these people view past photos at least 3 times is 0.76. For those who take at least 100 photos, the probability that they view past photos fewer than 3 times is 0.90. This information is shown in the tree diagram. The probability that a randomly chosen person views past photos fewer than 3 times is 0.801.

(i) Find x. [3]

(ii) Given that a person views past photos at least 3 times, find the probability that this person takes at least 100 photos. [4]

Answer: i) 0.15 ii) 0.427 **61/J15/4**

3 When Joanna cooks, the probability that the meal is served on time is $\frac{1}{5}$. The probability that the kitchen is left in a mess is $\frac{3}{5}$. The probability that the meal is not served on time and the kitchen is not left in a mess is $\frac{3}{10}$. Some of this information is shown in the following table.

	Kitchen left in a mess	Kitchen not left in a mess	Total
Meal served on time			$\frac{1}{5}$
Meal not served on time		$\frac{3}{10}$	
Total			1

(i) Copy and complete the table. [3]

(ii) Given that the kitchen is left in a mess, find the probability that the meal is not served on time. [2]

63/J15/2

Answer: i)

	Kitchen mess	Kitchen not mess	Total
On time	1/10	1/10	
Not on time	1/2		4/5
Total	3/5	4/10	

ii)5/6

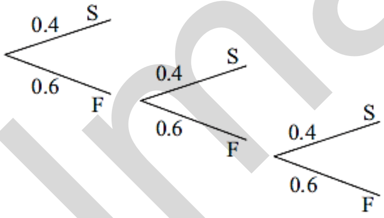
---

4 Nadia is very forgetful. Every time she logs in to her online bank she only has a 40% chance of remembering her password correctly. She is allowed 3 unsuccessful attempts on any one day and then the bank will not let her try again until the next day.

(i) Draw a fully labelled tree diagram to illustrate this situation. [3]

61/N15/6

Answer:



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5 The faces of a biased die are numbered 1, 2, 3, 4, 5 and 6. The probabilities of throwing odd numbers are all the same. The probabilities of throwing even numbers are all the same. The probability of throwing an odd number is twice the probability of throwing an even number.

(i) Find the probability of throwing a 3. [3]

(ii) The die is thrown three times. Find the probability of throwing two 5s and one 4. [3]

Answer: i) 2/9 ii)4/243

61/N15/7

6 In country X, 25% of people have fair hair. In country Y, 60% of people have fair hair. There are 20 million people in country X and 8 million people in country Y. A person is chosen at random from these 28 million people.

(i) Find the probability that the person chosen is from country X. [1]

(ii) Find the probability that the person chosen has fair hair. [2]

(iii) Find the probability that the person chosen is from country X, given that the person has fair hair. [2]

Answer: i) 5/7 ii)7/20 ii) 0.510

---

7 Ellie throws two fair tetrahedral dice, each with faces numbered 1, 2, 3 and 4. She notes the numbers on the faces that the dice land on. Event S is 'the sum of the two numbers is 4'. Event T is 'the product of the two numbers is an odd number'.

(i) Determine whether events S and T are independent, showing your working. [5]

(ii) Are events S and T exclusive? Justify your answer. [1]

Answer: i) Not independent ii)Not exclusive **63/N15/3**

---

8 Sharik attempts a multiple choice revision question on-line. There are 3 suggested answers, one of which is correct. When Sharik chooses an answer the computer indicates whether the answer is right or wrong. Sharik first chooses one of the three suggested answers at random. If this answer is wrong he has a second try, choosing an answer at random from the remaining 2. If this answer is also wrong Sharik then chooses the remaining answer, which must be correct.

(i) Draw a fully labelled tree diagram to illustrate the various choices that Sharik can make until the computer indicates that he has answered the question correctly. [4]

Answer: Diagram **62/N14/4**

---

9 Jodie tosses a biased coin and throws two fair tetrahedral dice. The probability that the coin shows a head is $\frac{1}{3}$. Each of the dice has four faces, numbered 1, 2, 3 and 4. Jodie's score is calculated from the numbers on the faces that the dice land on, as follows:

- if the coin shows a head, the two numbers from the dice are added together;
- if the coin shows a tail, the two numbers from the dice are multiplied together.

Find the probability that the coin shows a head given that Jodie's score is 8. [5]

Answer: 1/5 **61/N14/3**

10 Tom and Ben play a game repeatedly. The probability that Tom wins any game is 0.3. Each game is won by either Tom or Ben. Tom and Ben stop playing when one of them (to be called the champion) has won two games.

(i) Find the probability that Ben becomes the champion after playing exactly 2 games. [1]

(ii) Find the probability that Ben becomes the champion. [3]

(iii) Given that Tom becomes the champion, find the probability that he won the 2nd game. [4]

Answer: i) 0.49, ii) 0.784 **63/J14/6**

---

11 Playground equipment consists of swings (S), roundabouts (R), climbing frames (C) and play-houses (P). The numbers of pieces of equipment in each of 3 playgrounds are as follows.

Playground X	Playground Y	Playground Z
$3S$, $2R$, $4P$	$6S$, $3R$, $1C$, $2P$	$8S$, $3R$, $4C$, $1P$

Each day Nur takes her child to one of the playgrounds. The probability that she chooses playground X is $\frac{1}{4}$. The probability that she chooses playground Y is $\frac{1}{4}$. The probability that she chooses playground Z is $\frac{1}{2}$. When she arrives at the playground, she chooses one piece of equipment at random.

(i) Find the probability that Nur chooses a play-house. [4]

(ii) Given that Nur chooses a climbing frame, find the probability that she chose playground Y. [4]

Answer: i) 0.184, ii) 1/7 **61/J14/5**

---

12 The people living in two towns, Mumbok and Bagville, are classified by age. The numbers in thousands living in each town are shown in the table below.

	Mumbok	Bagville
Under 18 years	15	35
18 to 60 years	55	95
Over 60 years	20	30

One of the towns is chosen. The probability of choosing Mumbok is 0.6 and the probability of choosing Bagville is 0.4. Then a person is chosen at random from that town. Given that the person chosen is between 18 and 60 years old, find the probability that the town chosen was Mumbok. [5]

Answer: 0.607 **61/N13/2**

13 Q is the event 'Nicola throws two fair dice and gets a total of 5'. S is the event 'Nicola throws two fair dice and gets one low score (1, 2 or 3) and one high score (4, 5 or 6)'. Are events Q and S independent? Justify your answer. [4]

Answer: Independent **63/J13/1**

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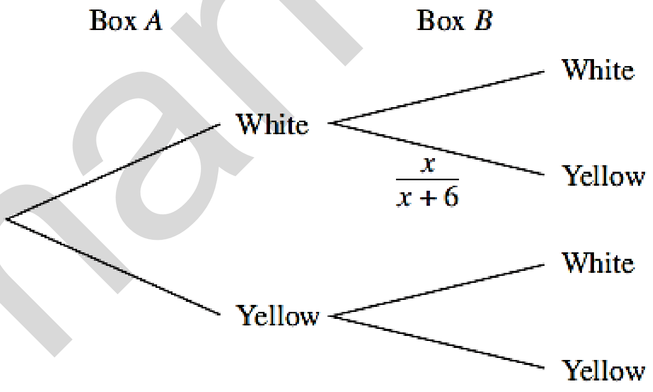
14 **(a)** John plays two games of squash. The probability that he wins his first game is 0.3. If he wins his first game, the probability that he wins his second game is 0.6. If he loses his first game, the probability that he wins his second game is 0.15. Given that he wins his second game, find the probability that he won his first game. [4]

(b) Jack has a pack of 15 cards. 10 cards have a picture of a robot on them and 5 cards have a picture of an aeroplane on them. Emma has a pack of cards. 7 cards have a picture of a robot on them and $x - 3$ cards have a picture of an aeroplane on them. One card is taken at random from Jack's pack and one card is taken at random from Emma's pack. The probability that both cards have pictures of robots on them is $\frac{7}{18}$. Write down an equation in terms of x and hence find the value of x. [4]

Answer: i) 0.632 ii) 8 **63/J13/5**

---

15 Box A contains 8 white balls and 2 yellow balls. Box B contains 5 white balls and x yellow balls. A ball is chosen at random from box A and placed in box B. A ball is then chosen at random from box B. The tree diagram below shows the possibilities for the colours of the balls chosen.



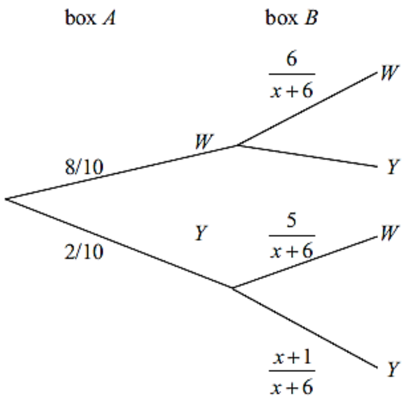
(i) Justify the probability $\frac{x}{x+6}$ on the tree diagram. [1]

(ii) Copy and complete the tree diagram. [4]

(iii) If the ball chosen from box A is white then the probability that the ball chosen from box B is also white is $\frac{1}{3}$. Show that the value of x is 12. [2]

(iv) Given that the ball chosen from box B is yellow, find the conditional probability that the ball chosen from box A was yellow. [4]

61/J13/7



Answer: **iv) 0.213**

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16 Ronnie obtained data about the gross domestic product (GDP) and the birth rate for 170 countries. He classified each GDP and each birth rate as either ‘low’, ‘medium’ or ‘high’. The table shows the number of countries in each category.

		Birth rate		
		Low	Medium	High
GDP	Low	3	5	45
	Medium	20	42	12
	High	35	8	0

One of these countries is chosen at random.

(i) Find the probability that the country chosen has a medium GDP. [1]

(ii) Find the probability that the country chosen has a low birth rate, given that it does not have a medium GDP. [2]

(iii) State with a reason whether or not the events ‘the country chosen has a high GDP’ and ‘the country chosen has a high birth rate’ are exclusive. [2]

One country is chosen at random from those countries which have a medium GDP and then a different country is chosen at random from those which have a medium birth rate.

(iv) Find the probability that both countries chosen have a medium GDP and a medium birth rate. [3]

Answer: i) 0.435 ii) 0.396 iii) Exclusive iv) 0.431 **63/N12/3**

17 Suzanne has 20 pairs of shoes, some of which have designer labels. She has 6 pairs of high-heeled shoes, of which 2 pairs have designer labels. She has 4 pairs of low-heeled shoes, of which 1 pair has designer labels. The rest of her shoes are pairs of sports shoes. Suzanne has 8 pairs of shoes with designer labels in total.

(i) Copy and complete the table below to show the number of pairs in each category. [2]

	Designer labels	No designer labels	Total
High-heeled shoes			
Low-heeled shoes			
Sports shoes			
Total			20

Suzanne chooses 1 pair of shoes at random to wear.

(ii) Find the probability that she wears the pair of low-heeled shoes with designer labels. [1]

(iii) Find the probability that she wears a pair of sports shoes. [1]

(iv) Find the probability that she wears a pair of high-heeled shoes, given that she wears a pair of shoes with designer labels. [1]

(v) State with a reason whether the events 'Suzanne wears a pair of shoes with designer labels' and 'Suzanne wears a pair of sports shoes' are independent. [2]

Answer: ii) 0.05 iii) 0.5 iv) 0.25 v) **63/J12/5**

---

18 Maria has 3 pre-set stations on her radio. When she switches her radio on, there is a probability of 0.3 that it will be set to station 1, a probability of 0.45 that it will be set to station 2 and a probability of 0.25 that it will be set to station 3. On station 1 the probability that the presenter is male is 0.1, on station 2 the probability that the presenter is male is 0.85 and on station 3 the probability that the presenter is male is p. When Maria switches on the radio, the probability that it is set to station 3 and the presenter is male is 0.075.

(i) Show that the value of p is 0.3. [1]

(ii) Given that Maria switches on and hears a male presenter, find the probability that the radio was set to station 2. [4]

Answer: ii) 0.785 **61/J12/2**

19 In a group of 30 teenagers, 13 of the 18 males watch 'Kops are Kids' on television and 3 of the 12 females watch 'Kops are Kids'.

(i) Find the probability that a person chosen at random from the group is either female or watches 'Kops are Kids' or both. [4]

(ii) Showing your working, determine whether the events 'the person chosen is male' and 'the person chosen watches Kops are Kids' are independent or not. [2]

Answer: i) 0.833 ii) not independent **63/N11/2**

---

20 A factory makes a large number of ropes with lengths either 3 m or 5 m. There are four times as many ropes of length 3 m as there are ropes of length 5 m.

(i) One rope is chosen at random. Find the expectation and variance of its length. [4]

(ii) Two ropes are chosen at random. Find the probability that they have different lengths. [2]

(iii) Three ropes are chosen at random. Find the probability that their total length is 11 m. [3]

Answer: i) 3.4, 0.64 ii) 0.32 iii) 0.384 **63/N11/3**

---

21 Bag A contains 4 balls numbered 2, 4, 5, 8. Bag B contains 5 balls numbered 1, 3, 6, 8, 8. Bag C contains 7 balls numbered 2, 7, 8, 8, 8, 8, 9. One ball is selected at random from each bag.

(i) Find the probability that exactly two of the selected balls have the same number. [5]

(ii) Given that exactly two of the selected balls have the same number, find the probability that they are both numbered 2. [2]

(iii) Event X is 'exactly two of the selected balls have the same number'. Event Y is 'the ball selected from bag A has number 2'. Showing your working, determine whether events X and Y are independent or not. [2]

Answer: i) 0.336 ii) 0.106 iii) not independent **61/N11/7**

---

22 Tim throws a fair die twice and notes the number on each throw.

(i) Tim calculates his final score as follows. If the number on the second throw is a 5 he multiplies the two numbers together, and if the number on the second throw is not a 5 he adds the two numbers together. Find the probability that his final score is

(a) 12, [1]

(b) 5. [3]

(ii) Events A, B, C are defined as follows.

A: the number on the second throw is 5
B: the sum of the numbers is 6
C: the product of the numbers is even

By calculation find which pairs, if any, of the events A, B and C are independent. [5]

Answer: ia) 1/36 b) 5/36 ii) None are independent **63/J11/4**

---

23 When Ted is looking for his pen, the probability that it is in his pencil case is 0.7. If his pen is in his pencil case he always finds it. If his pen is somewhere else, the probability that he finds it is 0.2. Given that Ted finds his pen when he is looking for it, find the probability that it was in his pencil case. [4]

Answer: 0.921 **61/J11/2**

---

24 **(b)** A bag contains 5 green balls and 3 yellow balls. Ronnie and Julie play a game in which they take turns to draw a ball from the bag at random without replacement. The winner of the game is the first person to draw a yellow ball. Julie draws the first ball. Find the probability that Ronnie wins the game. [4]

Answer: 0.393 **61/J11/7**

---

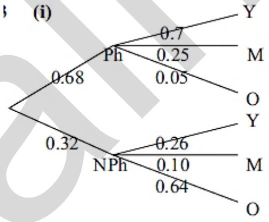
25 It was found that 68% of the passengers on a train used a cell phone during their train journey. Of those using a cell phone, 70% were under 30 years old, 25% were between 30 and 65 years old and the rest were over 65 years old. Of those not using a cell phone, 26% were under 30 years old and 64% were over 65 years old.

(i) Draw a tree diagram to represent this information, giving all probabilities as decimals. [2]

(ii) Given that one of the passengers is 45 years old, find the probability of this passenger using a cell phone during the journey. [3]

63/N10/3

Answer: **(i)**



ii) 0.842

26 Three friends, Rick, Brenda and Ali, go to a football match but forget to say which entrance to the ground they will meet at. There are four entrances, A, B, C and D. Each friend chooses an entrance independently.

- The probability that Rick chooses entrance A is $\frac{1}{3}$. The probabilities that he chooses entrances B, C or D are all equal.
- Brenda is equally likely to choose any of the four entrances.
- The probability that Ali chooses entrance C is $\frac{2}{7}$ and the probability that he chooses entrance D is $\frac{3}{5}$. The probabilities that he chooses the other two entrances are equal.

(i) Find the probability that at least 2 friends will choose entrance B. [4]

(ii) Find the probability that the three friends will all choose the same entrance. [4]

Answer: i) 0.0762 ii) 0.0571 **61/N10/5**

---

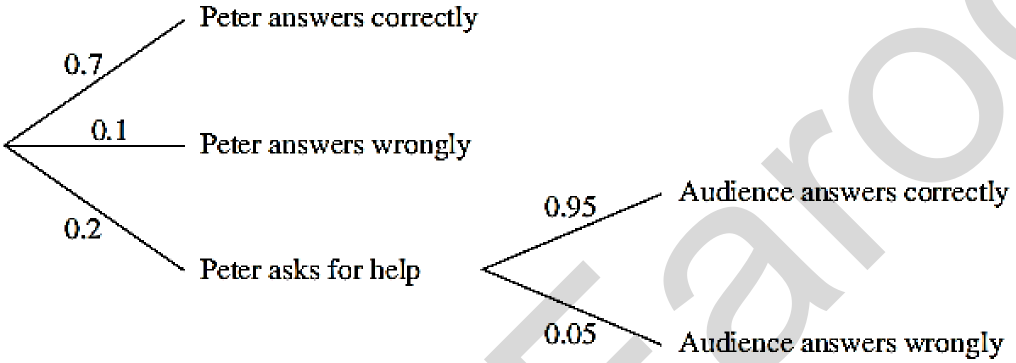
27 A bottle of sweets contains 13 red sweets, 13 blue sweets, 13 green sweets and 13 yellow sweets. 7 sweets are selected at random. Find the probability that exactly 3 of them are red. [3]

Answer: 0.176 **63/J10/1**

28 In a television quiz show Peter answers questions one after another, stopping as soon as a question is answered wrongly.

- The probability that Peter gives the correct answer himself to any question is 0.7.
- The probability that Peter gives a wrong answer himself to any question is 0.1.
- The probability that Peter decides to ask for help for any question is 0.2.

On the first occasion that Peter decides to ask for help he asks the audience. The probability that the audience gives the correct answer to any question is 0.95. This information is shown in the tree diagram below.



(i) Show that the probability that the first question is answered correctly is 0.89. [1]

On the second occasion that Peter decides to ask for help he phones a friend. The probability that his friend gives the correct answer to any question is 0.65.

(ii) Find the probability that the first two questions are both answered correctly. [6]

(iii) Given that the first two questions were both answered correctly, find the probability that Peter asked the audience. [3]

Answer: ii) 0.781 iii) 0.372 **61/J10/7**

---

29 The probability that the school bus is on time on any particular day is 0.6. If the bus is on time the probability that Sam the driver gets a cup of coffee is 0.9. If the bus is not on time the probability that Sam gets a cup of coffee is 0.3.

(i) Find the probability that Sam gets a cup of coffee. [2]

(ii) Given that Sam does not get a cup of coffee, find the probability that the bus is not on time. [3]

Answer: (i) 0.66 (ii) 0.824 **J16/61/Q3**

30 In a group of 30 adults, 25 are right-handed and 8 wear spectacles. The number who are right-handed and do not wear spectacles is 19.

(i) Copy and complete the following table to show the number of adults in each category. [2]

	Wears spectacles	Does not wear spectacles	Total
Right-handed			
Not right-handed			
Total			30

An adult is chosen at random from the group. Event X is 'the adult chosen is right-handed'; event Y is 'the adult chosen wears spectacles'.

(ii) Determine whether X and Y are independent events, justifying your answer. [3]

Answer: (i) 6 19 25 (ii) Not independent
2 3 5
8 22

J16/63/Q1

---

31 Ashfaq throws two fair dice and notes the numbers obtained. R is the event 'The product of the two numbers is 12'. T is the event 'One of the numbers is odd and one of the numbers is even'. By finding appropriate probabilities, determine whether events R and T are independent. [5]

Answer: Yes, events R and T are independent.

J17/61/Q2

---

32 Redbury United soccer team play a match every week. Each match can be won, drawn or lost. At the beginning of the soccer season the probability that Redbury United win their first match is $\frac{3}{5}$, with equal probabilities of losing or drawing. If they win the first match, the probability that they win the second match is $\frac{7}{10}$ and the probability that they lose the second match is $\frac{1}{10}$. If they draw the first match they are equally likely to win, draw or lose the second match. If they lose the first match, the probability that they win the second match is $\frac{3}{10}$ and the probability that they draw the second match is $\frac{1}{20}$.

(i) Draw a fully labelled tree diagram to represent the first two matches played by Redbury United in the soccer season. [2]

(ii) Given that Redbury United win the second match, find the probability that they lose the first match. [4]

Answer: (ii) 9 / 82

J17/61/Q3

33 A biased die has faces numbered 1 to 6. The probabilities of the die landing on 1, 3 or 5 are each equal to 0.1. The probabilities of the die landing on 2 or 4 are each equal to 0.2. The die is thrown twice. Find the probability that the sum of the numbers it lands on is 9. [4]

Answer: 0.1 **J17/63/Q1**

---

34 A shop sells two makes of coffee, Café Premium and Café Standard. Both coffees come in two sizes, large jars and small jars. Of the jars on sale, 65% are Café Premium and 35% are Café Standard. Of the Café Premium, 40% of the jars are large and of the Café Standard, 25% of the jars are large. A jar is chosen at random.

(i) Find the probability that the jar is small. [2]

(ii) Find the probability that the jar is Café Standard given that it is large. [3]

Answers: (i) 0.653 (ii) 0.252 **J17/63/Q3**

---

35 Deeti has 3 red pens and 1 blue pen in her left pocket and 3 red pens and 1 blue pen in her right pocket. 'Operation T' consists of Deeti taking one pen at random from her left pocket and placing it in her right pocket, then taking one pen at random from her right pocket and placing it in her left pocket.

(i) Find the probability that, when Deeti carries out operation T, she takes a blue pen from her left pocket and then a blue pen from her right pocket. [2]

The random variable X is the number of blue pens in Deeti's left pocket after carrying out operation T.

(ii) Find $P(X = 1)$. [3]

(iii) Given that the pen taken from Deeti's right pocket is blue, find the probability that the pen taken from Deeti's left pocket is blue. [4]

Answers: (i) 1/10 (ii) 14/20 (iii) 2/5 **N16/61/Q6**

---

36 A fair triangular spinner has three sides numbered 1, 2, 3. When the spinner is spun, the score is the number of the side on which it lands. The spinner is spun four times.

(i) Find the probability that at least two of the scores are 3. [3]

(ii) Find the probability that the sum of the four scores is 5. [3]

Answers: (i) 11/27 (ii) 4/81 **N16/63/Q2**

37 For a group of 250 cars the numbers, classified by colour and country of manufacture, are shown in the table.

	Germany	Japan	Korea
Silver	40	26	34
White	32	22	26
Red	28	12	30

One car is selected at random from this group. Find the probability that the selected car is

(i) a red or silver car manufactured in Korea, [1]

(ii) not manufactured in Japan. [1]

X is the event that the selected car is white. Y is the event that the selected car is manufactured in Germany.

(iii) By using appropriate probabilities, determine whether events X and Y are independent. [5]

Answers: (i) 64 / 250 (ii) 190 / 250 (iii) Independent **N16/63/Q4**

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38 Over a period of time Julian finds that on long-distance flights he flies economy class on 82% of flights. On the rest of the flights he flies first class. When he flies economy class, the probability that he gets a good night's sleep is x. When he flies first class, the probability that he gets a good night's sleep is 0.9.

(i) Draw a fully labelled tree diagram to illustrate this situation. [2]

The probability that Julian gets a good night's sleep on a randomly chosen flight is 0.285.

(ii) Find the value of x. [2]

(iii) Given that on a particular flight Julian does not get a good night's sleep, find the probability that he is flying economy class. [3]

Answers: (ii) 0.15 (ii) 0.975 **N17/61/Q5**

---

39 A statistics student asks people to complete a survey. The probability that a randomly chosen person agrees to complete the survey is 0.2. Find the probability that at least one of the first three people asked agrees to complete the survey. [2]

Answer: 0.488 **N17/63/Q1**

40 At the end of a revision course in mathematics, students have to pass a test to gain a certificate. The probability of any student passing the test at the first attempt is 0.85. Those students who fail are allowed to retake the test once, and the probability of any student passing the retake test is 0.65.

(i) Draw a fully labelled tree diagram to show all the outcomes. [2]

(ii) Given that a student gains the certificate, find the probability that this student fails the test on the first attempt. [4]

Answer: (ii) 0.103

N17/63/Q3

---

41 In a group of students, the numbers of boys and girls studying Art, Music and Drama are given in the following table. Each of these 160 students is studying exactly one of these subjects.

	Art	Music	Drama
Boys	24	40	32
Girls	15	12	37

(i) Find the probability that a randomly chosen student is studying Music. [1]

(ii) Determine whether the events 'a randomly chosen student is a boy' and 'a randomly chosen student is studying Music' are independent, justifying your answer. [2]

(iii) Find the probability that a randomly chosen student is not studying Drama, given that the student is a girl. [2]

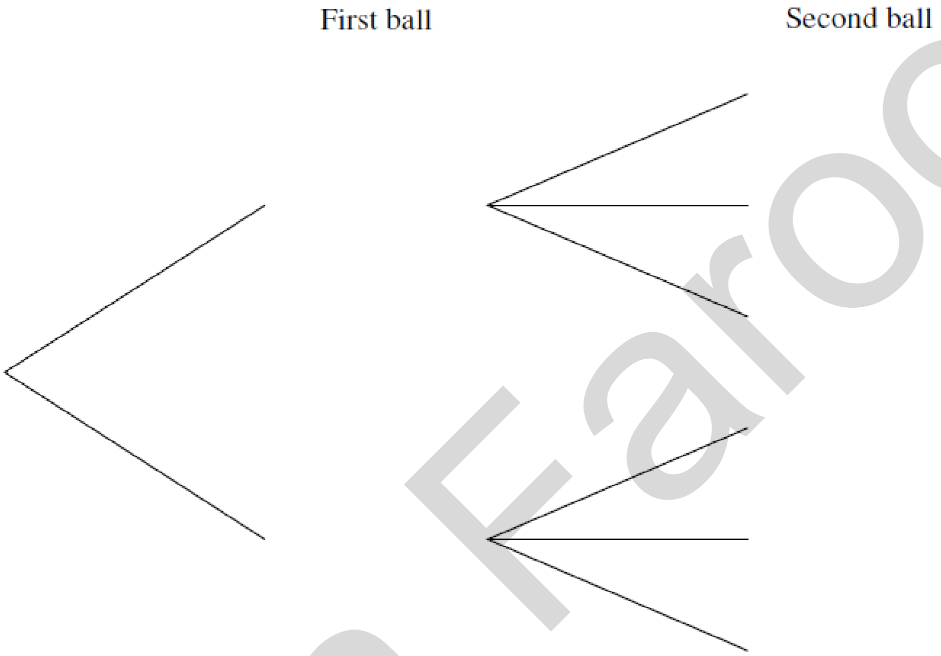
(iv) Three students are chosen at random. Find the probability that exactly 1 is studying Music and exactly 2 are boys. [5]

Answers: (i) $\frac{13}{40}$ (ii) Not independent (iii) $\frac{27}{64}$ (iv) 0.201

N18/61/Q7

42 A box contains 3 red balls and 5 blue balls. One ball is taken at random from the box and not replaced. A yellow ball is then put into the box. A second ball is now taken at random from the box.

(i) Complete the tree diagram to show all the outcomes and the probability for each branch. [2]

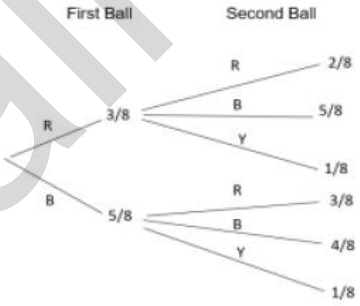


(ii) Find the probability that the two balls taken are the same colour. [2]

(iii) Find the probability that the first ball taken is red, given that the second ball taken is blue. [3]

Answer:

N18/63/Q3



Answer: $\frac{3}{7}$ (0.429) *Answer:* $\frac{13}{32}$ (0.406)

PROBABILITY DISTRIBUTIONS

A **random variable** is a quantity that has its **value** determined by the outcome of a random event.

Random variables can arise from **probability experiments**. For example, when you throw two dice,

X = the sum of the scores

is a random variable.

Similarly,

Y = product of the scores, and

Z = larger of the scores

are also random variables.

Example 3.1

A fair square spinner with sides labelled 1, 2, 3 and 4 is spun twice. The two scores obtained are added together to give the total, X. Draw up the probability distribution table for X.

Example 3.2

A box contains six black pens and four red pens. Three pens are taken at random from the box.

i. By considering the selection of pens without replacement, illustrate the various outcomes on a probability tree diagram

ii. The random variable X represents the number of red pens obtained. Find the probability distribution of X.

Example 3.3

A vegetable basket contains 12 peppers, of which 3 are red, 4 are green and 5 are yellow. Three peppers are taken, at random and without replacement, from the basket.

i. Find the probability that the three peppers are all different colours.

ii. Show that the probability that exactly 2 of the peppers taken are green is $\frac{12}{55}$

iii. The number of green peppers taken is denoted by the discrete random variable X. Draw up a probability distribution table for X.

Example 3.4

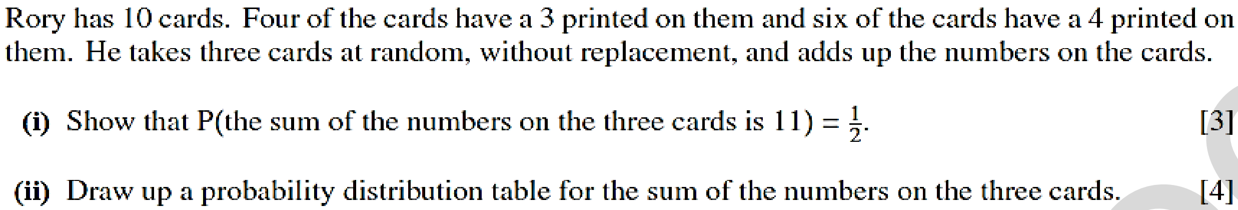
The following table shows the probability distribution for the random variable V.

v	2	3	4	5	6
$\mathrm{P}(V = v)$	0.05	c^2	$c + 0.1$	$2c + 0.05$	0.16

Find the value of the constant c and find $\mathrm{P}(V > 4)$.

Example 3.5

N13/62/Q7



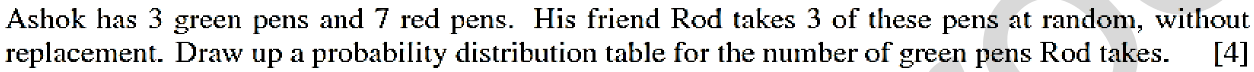
Rory has 10 cards. Four of the cards have a 3 printed on them and six of the cards have a 4 printed on them. He takes three cards at random, without replacement, and adds up the numbers on the cards.

(i) Show that P(the sum of the numbers on the three cards is 11) = $\frac{1}{2}$. [3]

(ii) Draw up a probability distribution table for the sum of the numbers on the three cards. [4]

Example 3.6

N12/61/Q1



Ashok has 3 green pens and 7 red pens. His friend Rod takes 3 of these pens at random, without replacement. Draw up a probability distribution table for the number of green pens Rod takes. [4]

EXPECTATION E(X) AND STANDARD DEVIATION OF X, E(X)

E(X) is read as 'E of X' and it gives an average or typical value of X, known as the expected value or expectation of X. This comparable with the mean in descriptive statistics.

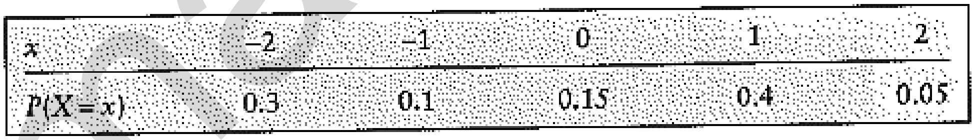
The expectation or expected mean can be thought of as the average value when the number of experiments increases indefinitely.

In a statistical experiment

- A practical approach results in frequency distribution and a mean value,
- A theoretical approach results in a probability distribution and an expected value, known as the expectation.

Example 3.7

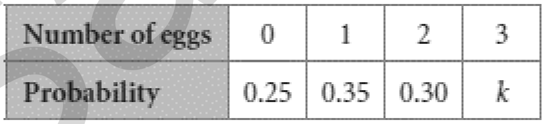
A random variable X has probability distribution and an expected value known as the expectation.



x	−2	−1	0	1	2
$P(X=x)$	0.3	0.1	0.15	0.4	0.05

Example 3.8

Birds of a particular species lay either 0, 1, 2 or 3 eggs in their nests with probabilities as shown in the table



Number of eggs	0	1	2	3
Probability	0.25	0.35	0.30	k

Find

i. the value of k

ii. the expected number of eggs laid in a nest

iii. the standard deviation of the number of eggs laid in a nest

Example 3.9

The probability distribution of the discrete random variable X is shown in the table below

x	-3	-1	0	4
$\mathrm{P}(X=x)$	a	b	0.15	0.4

Given that E(X)=0.75, find the values of a and b

Example 3.10

Every day Eduardo tries to phone his friend. Every time he phones there is a 50% chance that his friend will answer. If his friend answers, Eduardo does not phone again on that day. If his friend does not answer, Eduardo tries again in a few minutes' time. If his friend has not answered after 4 attempts, Eduardo does not try again on that day.

i. Draw a tree diagram to illustrate this situation.

ii. Let X be the number of unanswered phone calls made by Eduardo on a day. Copy and complete the table showing the probability distribution of X.

x	0	1	2	3	4
$\mathrm{P}(X=x)$		$\frac{1}{4}$			

iii. Calculate the expected number of unanswered phone calls on a day.

Example 3.11

J14/62/Q4

Coin A is weighted so that the probability of throwing a head is $\frac{2}{3}$. Coin B is weighted so that the probability of throwing a head is $\frac{1}{4}$. Coin A is thrown twice and coin B is thrown once.

(i) Show that the probability of obtaining exactly 1 head and 2 tails is $\frac{13}{36}$. [3]

(ii) Draw up the probability distribution table for the number of heads obtained. [4]

(iii) Find the expectation of the number of heads obtained. [2]

HOMEWORK:DISCRETE RANDOM VARIABLE– VARIANT 62

1 A box contains 10 pens of which 3 are new. A random sample of two pens is taken.

(i) Show that the probability of getting exactly one new pen in the sample is $\frac{7}{15}$. [2]

(ii) Construct a probability distribution table for the number of new pens in the sample. [3]

(iii) Calculate the expected number of new pens in the sample. [1]

Answers: (ii) $P(0) = \frac{7}{15}$, $P(1) = \frac{7}{15}$, $P(2) = \frac{1}{15}$; (ii) $\frac{3}{5}$.

J03/Q2

---

2 Two fair dice are thrown. Let the random variable X be the smaller of the two scores if the scores are different, or the score on one of the dice if the scores are the same.

(i) Copy and complete the following table to show the probability distribution of X. [3]

x	1	2	3	4	5	6
$P(X = x)$						

(ii) Find $E(X)$. [2]

Answers: (i)

x	1	2	3	4	5	6
$P(X = x)$	$\frac{11}{36}$	$\frac{9}{36}$	$\frac{7}{36}$	$\frac{5}{36}$	$\frac{3}{36}$	$\frac{1}{36}$

(ii) $E(X) = \frac{91}{36} = 2.53$.

J04/Q3

---

3 A fair dice has four faces. One face is coloured pink, one is coloured orange, one is coloured green and one is coloured black. Five such dice are thrown and the number that fall on a green face are counted. The random variable X is the number of dice that fall on a green face.

(i) Show that the probability of 4 dice landing on a green face is 0.0146, correct to 4 decimal places. [2]

(ii) Draw up a table for the probability distribution of X, giving your answers correct to 4 decimal places. [5]

Answer: (ii) 0, 0.2373; 1, 0.3955; 2, 0.2637; 3, 0.0879; 4, 0.0146; 5, 0.0010.

J05/Q3

4 A vegetable basket contains 12 peppers, of which 3 are red, 4 are green and 5 are yellow. Three peppers are taken, at random and without replacement, from the basket.

(i) Find the probability that the three peppers are all different colours. [3]

(ii) Show that the probability that exactly 2 of the peppers taken are green is $\frac{12}{55}$. [2]

(iii) The number of **green** peppers taken is denoted by the discrete random variable X. Draw up a probability distribution table for X. [5]

Answers: **(i)** $\frac{3}{11}$; **(iii)**

x	0	1	2	3
$P(X = x)$	$\frac{14}{55}$	$\frac{28}{55}$	$\frac{12}{55}$	$\frac{1}{55}$

J07/Q7

---

5 Every day Eduardo tries to phone his friend. Every time he phones there is a 50% chance that his friend will answer. If his friend answers, Eduardo does not phone again on that day. If his friend does not answer, Eduardo tries again in a few minutes' time. If his friend has not answered after 4 attempts, Eduardo does not try again on that day.

(i) Draw a tree diagram to illustrate this situation. [3]

(ii) Let X be the number of unanswered phone calls made by Eduardo on a day. Copy and complete the table showing the probability distribution of X. [4]

x	0	1	2	3	4
$P(X = x)$		$\frac{1}{4}$			

(iii) Calculate the expected number of unanswered phone calls on a day. [2]

Answers: (ii) $\frac{1}{2}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{16}$; (iii) $\frac{15}{16}$.

J08/Q6

6 Gohan throws a fair tetrahedral die with faces numbered 1, 2, 3, 4. If she throws an even number then her score is the number thrown. If she throws an odd number then she throws again and her score is the sum of both numbers thrown. Let the random variable X denote Gohan's score.

(i) Show that $\mathrm{P}(X = 2) = \frac{5}{16}$. [2]

(ii) The table below shows the probability distribution of X.

x	2	3	4	5	6	7
$\mathrm{P}(X = x)$	$\frac{5}{16}$	$\frac{1}{16}$	$\frac{3}{8}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{16}$

Calculate $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. [4]

Answers: (ii) 3.75, 2.19.

J09/Q2

---

7 A small farm has 5 ducks and 2 geese. Four of these birds are to be chosen at random. The random variable X represents the number of geese chosen.

(i) Draw up the probability distribution of X. [3]

(ii) Show that $\mathrm{E}(X) = \frac{8}{7}$ and calculate $\mathrm{Var}(X)$. [3]

(iii) When the farmer's dog is let loose, it chases either the ducks with probability $\frac{3}{5}$ or the geese with probability $\frac{2}{5}$. If the dog chases the ducks there is a probability of $\frac{1}{10}$ that they will attack the dog. If the dog chases the geese there is a probability of $\frac{3}{4}$ that they will attack the dog. Given that the dog is not attacked, find the probability that it was chasing the geese. [4]

Answers: (i) 0, $\frac{1}{7}$; 1, $\frac{4}{7}$; 2, $\frac{2}{7}$; (ii) $\frac{20}{49}$; (iii) $\frac{5}{32}$.

J10/62/Q6

---

8 The discrete random variable X has the following probability distribution.

x	1	3	5	7
$\mathrm{P}(X = x)$	0.3	a	b	0.25

(i) Write down an equation satisfied by a and b. [1]

(ii) Given that $\mathrm{E}(X) = 4$, find a and b. [3]

Answers: (ii) $a = 0.15$, $b = 0.3$.

N02/Q1

9 A discrete random variable X has the following probability distribution.

x	1	2	3	4
$\mathrm{P}(X = x)$	$3c$	$4c$	$5c$	$6c$

(i) Find the value of the constant c. [2]

(ii) Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. [4]

(iii) Find $\mathrm{P}(X > \mathrm{E}(X))$. [2]

Answers: (i) $\frac{1}{18}$ or 0.0556; (ii) 2.78, 1.17; (iii) 0.611. **N03/Q8**

---

10 A box contains five balls numbered 1, 2, 3, 4, 5. Three balls are drawn randomly at the same time from the box.

(i) By listing all possible outcomes (123, 124, etc.), find the probability that the sum of the three numbers drawn is an odd number. [2]

The random variable L denotes the largest of the three numbers drawn.

(ii) Find the probability that L is 4. [1]

(iii) Draw up a table to show the probability distribution of L. [3]

(iv) Calculate the expectation and variance of L. [3]

Answers: (i) 0.4; (ii) 0.3; (iii) P(3) = 0.1, P(4) = 0.3, P(5) = 0.6; (iv) E(L) = 4.5, Var (L) = 0.45. **N04/Q6**

---

11 In a competition, people pay \$1 to throw a ball at a target. If they hit the target on the first throw they receive \$5. If they hit it on the second or third throw they receive \$3, and if they hit it on the fourth or fifth throw they receive \$1. People stop throwing after the first hit, or after 5 throws if no hit is made. Mario has a constant probability of $\frac{1}{5}$ of hitting the target on any throw, independently of the results of other throws.

(i) Mario misses with his first and second throws and hits the target with his third throw. State how much profit he has made. [1]

(ii) Show that the probability that Mario's profit is \$0 is 0.184, correct to 3 significant figures. [2]

(iii) Draw up a probability distribution table for Mario's profit. [3]

(iv) Calculate his expected profit. [2]

Answers: (i) \$2; (iii) 4, 0.2; 2, 0.288; 0, 0.184; –1, 0.328; (iv) \$1.05. **N05/Q6**

---

12 A box contains 300 discs of different colours. There are 100 pink discs, 100 blue discs and 100 orange discs. The discs of each colour are numbered from 0 to 99. Five discs are selected at random, one at a time, with replacement. Find

(i) the probability that no orange discs are selected, [1]

(ii) the probability that exactly 2 discs with numbers ending in a 6 are selected, [3]

(iii) the probability that exactly 2 orange discs with numbers ending in a 6 are selected, [2]

(iv) the mean and variance of the number of pink discs selected. [2]

Answers: (i) 0.132; (ii) 0.0729; (iii) 0.0100; (iv) $\frac{5}{3}$, $\frac{10}{9}$. **N05/Q5**

---

13 Box A contains 5 red paper clips and 1 white paper clip. Box B contains 7 red paper clips and 2 white paper clips. One paper clip is taken at random from box A and transferred to box B. One paper clip is then taken at random from box B.

(i) Find the probability of taking both a white paper clip from box A and a red paper clip from box B. [2]

(ii) Find the probability that the paper clip taken from box B is red. [2]

(iii) Find the probability that the paper clip taken from box A was red, given that the paper clip taken from box B is red. [2]

(iv) The random variable X denotes the number of times that a red paper clip is taken. Draw up a table to show the probability distribution of X. [4]

Answers: (i) $\frac{7}{60}$; (ii) $\frac{47}{60}$; (iii) $\frac{40}{47}$; (iv) 0, $\frac{3}{60}$; 1, $\frac{17}{60}$; 2, $\frac{40}{60}$. **N07/Q7**

14 A fair die has one face numbered 1, one face numbered 3, two faces numbered 5 and two faces numbered 6.

(i) Find the probability of obtaining at least 7 odd numbers in 8 throws of the die. [4]

The die is thrown twice. Let X be the sum of the two scores. The following table shows the possible values of X.

Second throw

		1	3	5	5	6	6
	1	2	4	6	6	7	7
	3	4	6	8	8	9	9
First throw	5	6	8	10	10	11	11
	5	6	8	10	10	11	11
	6	7	9	11	11	12	12
	6	7	9	11	11	12	12

(ii) Draw up a table showing the probability distribution of X. [3]

(iii) Calculate $\mathrm{E}(X)$. [2]

(iv) Find the probability that X is greater than $\mathrm{E}(X)$. [2]

Answers: (i) 0.195; (ii) 2, $\frac{1}{36}$; 4, $\frac{2}{36}$; 6, $\frac{5}{36}$; 7, $\frac{4}{36}$; 8, $\frac{4}{36}$; 9, $\frac{4}{36}$; 10, $\frac{4}{36}$; 11, $\frac{8}{36}$, 12, $\frac{4}{36}$; **N08/Q7**

(iii) $\frac{26}{3}$; (iv) $\frac{5}{9}$.

---

15 In a particular discrete probability distribution the random variable X takes the value $\frac{120}{r}$ with probability $\frac{r}{45}$, where r takes all integer values from 1 to 9 inclusive.

(i) Show that $\mathrm{P}(X = 40) = \frac{1}{15}$. [2]

(ii) Construct the probability distribution table for X. [3]

(iii) Which is the modal value of X? [1]

(iv) Find the probability that X lies between 18 and 100. [2]

Answers: (ii) 120, $\frac{1}{45}$; 60, $\frac{2}{45}$; 40, $\frac{3}{45}$; 30, $\frac{4}{45}$; 24, $\frac{5}{45}$; 20, $\frac{6}{45}$; 17.1, $\frac{7}{45}$; 15, $\frac{8}{45}$; 13.3, $\frac{9}{45}$; **N09/62/Q5**

(iii) $\frac{40}{3}$ or 13.3; (iv) $\frac{4}{9}$ or 0.444.

16 The discrete random variable X takes the values 1, 4, 5, 7 and 9 only. The probability distribution of X is shown in the table.

x	1	4	5	7	9
$P(X = x)$	$4p$	$5p^2$	$1.5p$	$2.5p$	$1.5p$

Find p. [3]

Answer: 0.1.

N10/62/Q1

---

17 The discrete random variable X has the following probability distribution.

x	0	1	2	3	4
$P(X = x)$	0.26	q	$3q$	0.05	0.09

(i) Find the value of q. [2]

(ii) Find $E(X)$ and $Var(X)$. [3]

N06/Q2

Answers: (i) 0.15; (ii) 1.56, 1.41.

---

18 The random variable X takes the values -2, 0 and 4 only. It is given that $P(X = -2) = 2p$, $P(X = 0) = p$ and $P(X = 4) = 3p$.

(i) Find p. [2]

(ii) Find $E(X)$ and $Var(X)$. [4]

N07/Q2

Answers: (i) $\frac{1}{6}$; (ii) $\frac{4}{3}$, $\frac{68}{9}$.

19 Judy and Steve play a game using five cards numbered 3, 4, 5, 8, 9. Judy chooses a card at random, looks at the number on it and replaces the card. Then Steve chooses a card at random, looks at the number on it and replaces the card. If their two numbers are equal the score is 0. Otherwise, the smaller number is subtracted from the larger number to give the score.

(i) Show that the probability that the score is 6 is 0.08. [1]

(ii) Draw up a probability distribution table for the score. [2]

(iii) Calculate the mean score. [1]

If the score is 0 they play again. If the score is 4 or more Judy wins. Otherwise Steve wins. They continue playing until one of the players wins.

(iv) Find the probability that Judy wins with the second choice of cards. [3]

(v) Find an expression for the probability that Judy wins with the *n*th choice of cards. [2]

Answers: (ii) 0, 0.2; 1, 0.24; 2, 0.08; 3, 0.08; 4, 0.16; 5, 0.16; 6, 0.08; (iii) 2.56; (iv) 0.0
(v) $(0.2)^{n-1} \times 0.4$

J11/62/Q7

---

20 The random variable X has the probability distribution shown in the table.

x	2	4	6
$\mathrm{P}(X = x)$	0.5	0.4	0.1

Two independent values of X are chosen at random. The random variable Y takes the value 0 if the two values of X are the same. Otherwise the value of Y is the larger value of X minus the smaller value of X.

(i) Draw up the probability distribution table for Y. [4]

(ii) Find the expected value of Y. [1]

Answers: (i) $\mathrm{P}(Y = 0) = 0.42$, $\mathrm{P}(Y = 2) = 0.48$, $\mathrm{P}(Y = 4) = 0.1$ (ii) $\mathrm{E}(Y) = 1.36$.

J12/62/Q2

---

21 Susan has a bag of sweets containing 7 chocolates and 5 toffees. Ahmad has a bag of sweets containing 3 chocolates, 4 toffees and 2 boiled sweets. A sweet is taken at random from Susan's bag and put in Ahmad's bag. A sweet is then taken at random from Ahmad's bag.

(i) Find the probability that the two sweets taken are a toffee from Susan's bag and a boiled sweet from Ahmad's bag. [2]

(ii) Given that the sweet taken from Ahmad's bag is a chocolate, find the probability that the sweet taken from Susan's bag was also a chocolate. [4]

(iii) The random variable X is the number of times a chocolate is taken. State the possible values of X and draw up a table to show the probability distribution of X. [5]

Answers: 1/12, 28/43, P(0) = 7/24, P(1) = 19/40, P(2) = 7/30.

J13/62/Q7

---

22 Coin A is weighted so that the probability of throwing a head is $\frac{2}{3}$. Coin B is weighted so that the probability of throwing a head is $\frac{1}{4}$. Coin A is thrown twice and coin B is thrown once.

(i) Show that the probability of obtaining exactly 1 head and 2 tails is $\frac{13}{36}$. [3]

(ii) Draw up the probability distribution table for the number of heads obtained. [4]

(iii) Find the expectation of the number of heads obtained. [2]

Answers: $P(0) = \frac{1}{12}$, $P(1) = \frac{13}{36}$, $P(2) = \frac{4}{9}$, $P(3) = \frac{1}{9}$; $\frac{19}{12}$

J14/62/Q4

---

23 A box contains 5 discs, numbered 1, 2, 4, 6, 7. William takes 3 discs at random, without replacement, and notes the numbers on the discs.

(i) Find the probability that the numbers on the 3 discs are two even numbers and one odd number. [3]

The smallest of the numbers on the 3 discs taken is denoted by the random variable S.

(ii) By listing all possible selections (126, 246 and so on) draw up the probability distribution table for S. [5]

Answers: (i) 0.6, (ii) P(1) = 0.6, P(2) = 0.3, P(4) = 0.1

J15/62/Q5

---

24 A fair tetrahedral die has four triangular faces, numbered 1, 2, 3 and 4. The score when this die is thrown is the number on the face that the die lands on. This die is thrown three times. The random variable X is the sum of the three scores.

(i) Show that $P(X = 9) = \frac{10}{64}$. [3]

(ii) Copy and complete the probability distribution table for X. [3]

x	3	4	5	6	7	8	9	10	11	12
$P(X = x)$	$\frac{1}{64}$	$\frac{3}{64}$			$\frac{12}{64}$					

(iii) Event R is 'the sum of the three scores is 9'. Event S is 'the product of the three scores is 16'. Determine whether events R and S are independent, showing your working. [5]

Answers: (i) 10/64; (ii) P(5, 6, 8, 9, 10, 11, 12) = (6/64, 10/64, 12/64, 10/64, 6/64, 3/64, 1/64);
(iii) Not independent.

N12/62/Q6

25 Rory has 10 cards. Four of the cards have a 3 printed on them and six of the cards have a 4 printed on them. He takes three cards at random, without replacement, and adds up the numbers on the cards.

(i) Show that P(the sum of the numbers on the three cards is 11) = $\frac{1}{2}$. [3]

(ii) Draw up a probability distribution table for the sum of the numbers on the three cards. [4]

Event R is 'the sum of the numbers on the three cards is 11'. Event S is 'the number on the first card taken is a 3'.

(iii) Determine whether events R and S are independent. Justify your answer. [3]

(iv) Determine whether events R and S are exclusive. Justify your answer. [1]

Answers: **(ii)** P(9) = 1/30, P(10) = 3/10, P(11) = ½, P(12) = 1/6.

N13/62/Q7

26 Sharik attempts a multiple choice revision question on-line. There are 3 suggested answers, one of which is correct. When Sharik chooses an answer the computer indicates whether the answer is right or wrong. Sharik first chooses one of the three suggested answers at random. If this answer is wrong he has a second try, choosing an answer at random from the remaining 2. If this answer is also wrong Sharik then chooses the remaining answer, which must be correct.

(i) Draw a fully labelled tree diagram to illustrate the various choices that Sharik can make until the computer indicates that he has answered the question correctly. [4]

(ii) The random variable X is the number of attempts that Sharik makes up to and including the one that the computer indicates is correct. Draw up the probability distribution table for X and find E(X). [4]

N14/62/Q4

27 A fair spinner A has edges numbered 1, 2, 3, 3. A fair spinner B has edges numbered −3, −2, −1, 1. Each spinner is spun. The number on the edge that the spinner comes to rest on is noted. Let X be the sum of the numbers for the two spinners.

(i) Copy and complete the table showing the possible values of X. [1]

Spinner A

		1	2	3	3
Spinner B	−3	−2			
	−2			1	
	−1				
	1				

(ii) Draw up a table showing the probability distribution of X. [3]

(iii) Find Var(X). [3]

(iv) Find the probability that X is even, given that X is positive. [2]

Answers: **(iii)** 23/8, **(iv)** 5/9

N15/62/Q6

HOMEWORK:DISCRETE RANDOM VARIABLE VARIANT 61 AND 63

1 A pet shop has 9 rabbits for sale, 6 of which are white. A random sample of two rabbits is chosen without replacement.

(i) Show that the probability that exactly one of the two rabbits in the sample is white is $\frac{1}{2}$. [2]

(ii) Construct the probability distribution table for the number of white rabbits in the sample. [3]

(iii) Find the expected value of the number of white rabbits in the sample. [1]

63/J15/4

Answer i) 0.5 ii)

x	0	1	2
Prob	1/12	1/2	5/12

iii)1.33

2 Nadia is very forgetful. Every time she logs in to her online bank she only has a 40% chance of remembering her password correctly. She is allowed 3 unsuccessful attempts on any one day and then the bank will not let her try again until the next day.

(i) Draw a fully labelled tree diagram to illustrate this situation. [3]

(ii) Let X be the number of unsuccessful attempts Nadia makes on any day that she tries to log in to her bank. Copy and complete the following table to show the probability distribution of X. [4]

x	0	1	2	3
$P(X = x)$		0.24		

(iii) Calculate the expected number of unsuccessful attempts made by Nadia on any day that she tries to log in. [2]

Answer: i) ii)0.4, 0.144, 0.216 **61/N15/6**

3 A box contains 2 green apples and 2 red apples. Apples are taken from the box, one at a time, without replacement. When both red apples have been taken, the process stops. The random variable X is the number of apples which have been taken when the process stops.

(i) Show that $P(X = 3) = \frac{1}{3}$. [3]

(ii) Draw up the probability distribution table for X. [3]

Another box contains 2 yellow peppers and 5 orange peppers. Three peppers are taken at random from the box without replacement.

(iii) Given that at least 2 of the peppers taken from the box are orange, find the probability that all 3 peppers are orange. [5]

63/N14/7

Answer: ii)

X	2	3	4
Prob	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$

iii) 0.333

4 Sharik attempts a multiple choice revision question on-line. There are 3 suggested answers, one of which is correct. When Sharik chooses an answer the computer indicates whether the answer is right or wrong. Sharik first chooses one of the three suggested answers at random. If this answer is wrong he has a second try, choosing an answer at random from the remaining 2. If this answer is also wrong Sharik then chooses the remaining answer, which must be correct.

(i) Draw a fully labelled tree diagram to illustrate the various choices that Sharik can make until the computer indicates that he has answered the question correctly. [4]

(ii) The random variable X is the number of attempts that Sharik makes up to and including the one that the computer indicates is correct. Draw up the probability distribution table for X and find E(X). [4]

62/N14/4

Answer: ii)

x	1	2	3
Prob	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

---

5 The number of phone calls, X, received per day by Sarah has the following probability distribution.

x	0	1	2	3	4	$\geqslant 5$
$P(X = x)$	0.24	0.35	$2k$	k	0.05	0

(i) Find the value of k. [2]

(ii) Find the mode of X. [1]

(iii) Find the probability that the number of phone calls received by Sarah on any particular day is more than the mean number of phone calls received per day. [3]

Answer: i) 0.12 ii) 1 iii) 0.41 **61/N14/2**

---

6 A pet shop has 6 rabbits and 3 hamsters. 5 of these pets are chosen at random. The random variable X represents the number of hamsters chosen.

(i) Show that the probability that exactly 2 hamsters are chosen is $\frac{10}{21}$. [2]

(ii) Draw up the probability distribution table for X. [4]

63/J14/3

Answer:

x	0	1	2	3
Prob	2/42	15/42	20/42	5/42

7 A book club sends 6 paperback and 2 hardback books to Mrs Hunt. She chooses 4 of these books at random to take with her on holiday. The random variable X represents the number of paperback books she chooses.

(i) Show that the probability that she chooses exactly 2 paperback books is $\frac{3}{14}$. [2]

(ii) Draw up the probability distribution table for X. [3]

(iii) You are given that $\mathrm{E}(X) = 3$. Find $\mathrm{Var}(X)$. [2]

61/J14/4

Answer: ii)

x	2	3	4
Prob	3/14	8/14	3/14

iii) 0.429

---

8 Dayo chooses two digits at random, without replacement, from the 9-digit number 113 333 555.

(i) Find the probability that the two digits chosen are equal. [3]

(ii) Find the probability that one digit is a 5 and one digit is not a 5. [3]

(iii) Find the probability that the first digit Dayo chose was a 5, given that the second digit he chose is not a 5. [4]

(iv) The random variable X is the number of 5s that Dayo chooses. Draw up a table to show the probability distribution of X. [3]

63/N13/7

Answer: i) 0.278 ii) 0.5 iii) 0.375 iv)

x	0	1	2
$P(X = x)$	5/12	1/2	1/12

9 James has a fair coin and a fair tetrahedral die with four faces numbered 1, 2, 3, 4. He tosses the coin once and the die twice. The random variable X is defined as follows.

- If the coin shows a **head** then X is the **sum** of the scores on the two throws of the die.
- If the coin shows a **tail** then X is the score on the **first throw** of the die only.

(i) Explain why $X = 1$ can only be obtained by throwing a tail, and show that $P(X = 1) = \frac{1}{8}$. [2]

(ii) Show that $P(X = 3) = \frac{3}{16}$. [4]

(iii) Copy and complete the probability distribution table for X. [3]

x	1	2	3	4	5	6	7	8
$P(X = x)$	$\frac{1}{8}$		$\frac{3}{16}$		$\frac{1}{8}$		$\frac{1}{16}$	$\frac{1}{32}$

Event Q is 'James throws a tail'. Event R is 'the value of X is 7'.

(iv) Determine whether events Q and R are exclusive. Justify your answer. [2]

61/N13/7

Answer: iii)

X	1	2	3	4	5	6	7	8
Prob		5/32		7/32		3/32		

iv) Yes

---

10 The discrete random variable X has the following probability distribution.

x	-3	0	2	4
$P(X = x)$	p	q	r	0.4

Given that $E(X) = 2.3$ and $Var(X) = 3.01$, find the values of p, q and r. [6]

Answer: 0.033, 0.4, 0.167 **63/N12/2**

---

11 Ashok has 3 green pens and 7 red pens. His friend Rod takes 3 of these pens at random, without replacement. Draw up a probability distribution table for the number of green pens Rod takes. [4]

61/N12/1

Answer:

$P(0) = 7/10 \times 6/9 \times 5/8 = 210/720$

$P(1) = 3/10 \times 7/9 \times 6/8 \times 3C1 = 378/720$

$P(2) = 3/10 \times 2/9 \times 7/8 \times 3C2 = 126/720$

$P(3) = 3/10 \times 2/9 \times 1/8 = 6/720\ (1/120)$

12 A company set up a display consisting of 20 fireworks. For each firework, the probability that it fails to work is 0.05, independently of other fireworks.

(i) Find the probability that more than 1 firework fails to work. [3]

The 20 fireworks cost the company \$24 each. 450 people pay the company \$10 each to watch the display. If more than 1 firework fails to work they get their money back.

(ii) Calculate the expected profit for the company. [4]

Answer: i) 0.264 ii) 2830 **61/N12/5**

---

13 The six faces of a fair die are numbered 1, 1, 1, 2, 3, 3. The score for a throw of the die, denoted by the random variable W, is the number on the top face after the die has landed.

(i) Find the mean and standard deviation of W. [3]

(ii) The die is thrown twice and the random variable X is the sum of the two scores. Draw up a probability distribution table for X. [4]

(iii) The die is thrown n times. The random variable Y is the number of times that the score is 3. Given that E(Y) = 8, find Var(Y). [3]

63/J12/4

Answer: i) 1.83 , 0.898

(ii)

x	2	3	4	5	6
Pr	9/36	6/36	13/36	4/36	4/36

iii) 5.33

---

14 A spinner has 5 sides, numbered 1, 2, 3, 4 and 5. When the spinner is spun, the score is the number of the side on which it lands. The score is denoted by the random variable X, which has the probability distribution shown in the table.

x	1	2	3	4	5
$P(X = x)$	0.3	0.15	$3p$	$2p$	0.05

(i) Find the value of p. [1]

A second spinner has 3 sides, numbered 1, 2 and 3. The score when this spinner is spun is denoted by the random variable Y. It is given that $P(Y = 1) = 0.3$, $P(Y = 2) = 0.5$ and $P(Y = 3) = 0.2$.

(ii) Find the probability that, when both spinners are spun together,

(a) the sum of the scores is 4, [3]

(b) the product of the scores is less than 8. [3]

Answer : i) 0.1 ii) 0.225 iii) 0.765 **61/J12/3**

15 A team of 4 is to be randomly chosen from 3 boys and 5 girls. The random variable X is the number of girls in the team.

(i) Draw up a probability distribution table for X. [4]

(ii) Given that $\mathrm{E}(X) = \frac{5}{2}$, calculate $\mathrm{Var}(X)$. [2]

Answer: ii) 15/28 **61/N11/3**

---

16 The probability that Sue completes a Sudoku puzzle correctly is 0.75.

(i) Sue attempts n Sudoku puzzles. Find the least value of n for which the probability that she completes all n puzzles correctly is less than 0.06. [3]

Sue attempts 14 Sudoku puzzles every month. The number that she completes successfully is denoted by X.

(ii) Find the value of X that has the highest probability. You may assume that this value is one of the two values closest to the mean of X. [3]

(iii) Find the probability that in exactly 3 of the next 5 months Sue completes more than 11 Sudoku puzzles correctly. [5]

Answer: i) 10 ii) 11 iii) 0.115 **63/J11/6**

---

17 The possible values of the random variable X are the 8 integers in the set $\{-2, -1, 0, 1, 2, 3, 4, 5\}$. The probability of X being 0 is $\frac{1}{10}$. The probabilities for all the other values of X are equal. Calculate

(i) $\mathrm{P}(X < 2)$, [2]

(ii) the variance of X, [3]

(iii) the value of a for which $\mathrm{P}(-a \leqslant X \leqslant 2a) = \frac{17}{35}$. [1]

Answer: i) 0.486 ii) 5.33 iii) 1 **61/J11/3**

---

18 In a probability distribution the random variable X takes the value x with probability kx, where x takes values 1, 2, 3, 4, 5 only.

(i) Draw up a probability distribution table for X, in terms of k, and find the value of k. [3]

(ii) Find $\mathrm{E}(X)$. [2]

63/N11/2

Answer:

x	1	2	3	4	5
Prob	k	$2k$	$3k$	$4k$	$5k$

ii) 3.67

19 Set A consists of the ten digits 0, 0, 0, 0, 0, 0, 2, 2, 2, 4.

Set B consists of the seven digits 0, 0, 0, 0, 2, 2, 2.

One digit is chosen at random from each set. The random variable X is defined as the sum of these two digits.

(i) Show that $P(X = 2) = \frac{3}{7}$. [2]

(ii) Tabulate the probability distribution of X. [2]

(iii) Find $E(X)$ and $Var(X)$. [3]

(iv) Given that $X = 2$, find the probability that the digit chosen from set A was 2. [2]

63/J10/5

Answer:

x	0	2	4	6
$P(X = x)$	24/70	30/70	13/70	3/70

iii) 13/7 and 2.78 iv) 0.4

---

20 Sanket plays a game using a biased die which is twice as likely to land on an even number as on an odd number. The probabilities for the three even numbers are all equal and the probabilities for the three odd numbers are all equal.

(i) Find the probability of throwing an odd number with this die. [2]

Sanket throws the die once and calculates his score by the following method.

- If the number thrown is 3 or less he multiplies the number thrown by 3 and adds 1.
- If the number thrown is more than 3 he multiplies the number thrown by 2 and subtracts 4.

The random variable X is Sanket's score.

(ii) Show that $P(X = 8) = \frac{2}{9}$. [2]

The table shows the probability distribution of X.

x	4	6	7	8	10
$P(X = x)$	$\frac{3}{9}$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{1}{9}$

(iii) Given that $E(X) = \frac{58}{9}$, find $Var(X)$. [2]

Sanket throws the die twice.

(iv) Find the probability that the total of the scores on the two throws is 16. [2]

(v) Given that the total of the scores on the two throws is 16, find the probability that the score on the first throw was 6. [3]

Answer: i) 1/3 iii) 4.02 iv) 0.0741 v) 1/6 **61/N10/7**

---

21 The probability distribution of the discrete random variable X is shown in the table below.

x	-3	-1	0	4
$P(X = x)$	a	b	0.15	0.4

Given that $E(X) = 0.75$, find the values of a and b. [4]

Answer: 0.2, 0.25 **61/J10/1**

---

22 The faces of a biased die are numbered 1, 2, 3, 4, 5 and 6. The random variable X is the score when the die is thrown. The following is the probability distribution table for X.

x	1	2	3	4	5	6
$P(X = x)$	p	p	p	p	0.2	0.2

The die is thrown 3 times. Find the probability that the score is 4 on not more than 1 of the 3 throws. [5]

Answer: 0.939 **J16/61/Q2**

---

23 A box contains 2 green sweets and 5 blue sweets. Two sweets are taken at random from the box, without replacement. The random variable X is the number of green sweets taken. Find $E(X)$ and $\mathrm{Var}(X)$. [6]

Answers: 4/7 (0.571), 50/147 (0.340) **J16/61/Q4**

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24 Two ordinary fair dice are thrown. The resulting score is found as follows.

- If the two dice show different numbers, the score is the smaller of the two numbers.
- If the two dice show equal numbers, the score is 0.

(i) Draw up the probability distribution table for the score. [4]

(ii) Calculate the expected score. [2]

Answer: **(i)** **J16/63/Q3**

x	0	1	2	3	4	5
$P(X = x)$	1/6	10/36	8/36	6/36	4/36	2/36

(ii) 35/18 or 1.94

25 Two fair six-sided dice with faces numbered 1, 2, 3, 4, 5, 6 are thrown and the two scores are noted. The difference between the two scores is defined as follows.

- If the scores are equal the difference is zero.
- If the scores are not equal the difference is the larger score minus the smaller score.

Find the expectation of the difference between the two scores. [5]

Answer: 1.94

N16/61/Q2

---

26 The discrete random variable X has the following probability distribution.

x	1	2	3	6
$\mathrm{P}(X = x)$	0.15	p	0.4	q

Given that $\mathrm{E}(X) = 3.05$, find the values of p and q. [4]

Answer: $p = 0.25$, $q = 0.2$

N17/61/Q1

---

27 A fair die with faces numbered 1, 2, 2, 2, 3, 6 is thrown. The score, X, is found by squaring the number on the face the die shows and then subtracting 4.

(i) Draw up a table to show the probability distribution of X. [3]

(ii) Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. [3]

N17/63/Q4

Answers: (i)

x	–3	0	5	32
Prob	1/6	1/2	1/6	1/6

(ii) 17/3, 144

28 A random variable X has the probability distribution shown in the following table, where p is a constant.

x	-1	0	1	2	4
$P(X = x)$	p	p	$2p$	$2p$	0.1

(i) Find the value of p. [1]

(ii) Given that $E(X) = 1.15$, find $Var(X)$. [2]

Answer: (i) 0.15 (ii) 1.93

N18/61/Q2

---

29 A fair 6-sided die has the numbers $-1, -1, 0, 0, 1, 2$ on its faces. A fair 3-sided spinner has edges numbered $-1, 0, 1$. The die is thrown and the spinner is spun. The number on the uppermost face of the die and the number on the edge on which the spinner comes to rest are noted. The sum of these two numbers is denoted by X.

(i) Draw up a table showing the probability distribution of X. [3]

(ii) Find $Var(X)$. [3]

N18/63/Q2

x	-2	-1	0	1	2	3
$P(X = x)$	$\frac{2}{18}$	$\frac{4}{18}$	$\frac{5}{18}$	$\frac{4}{18}$	$\frac{2}{18}$	$\frac{1}{18}$

Answer: $\frac{65}{36}$ (1.81)

THE BINOMIAL DISTRIBUTION

Conditions for a binomial model

For a situation to be described using a binomial model,

- A finite number, n, trials are carried out,
- The trials are independent,
- The outcome of each trial is deemed either a success or a failure,
- The probability, p, of a successful outcome is the same for each trial.

The discrete random variable, X , is the number of successful outcomes in n trials.

Example 4.1

At a Supermarket, 60% of customers pay by credit card. Find the probability that in a randomly selected sample of ten customers,

(a) exactly two pay by credit card,
(b) more than seven pay by credit card.

Example 4.2

J14/62/Q1

In a certain country 12% of houses have solar heating. 19 houses are chosen at random. Find the probability that fewer than 4 houses have solar heating. [4]

Example 4.3

The random variable X is distributed B (7, 0.2). Find, correct to three decimal places,

(a) $\mathrm{P}(X = 3)$
(b) $\mathrm{P}\ (1 < X < 4)$,
(c) $\mathrm{P}\ (X\ > 1)$

Example 4.4

Given that $X \sim \mathrm{B}(n, 0.4)$ and that $\mathrm{P}(X = 0) < 0.1$, find the least possible value of n.

Example 4.5

A box contains a large number of pens. The probability that a pen is faulty is 0.1
How many pens would you need to select to be more than 95% certain of picking at least one faulty one?

EXPECTATION AND VARIANCE OF THE BIONOMIAL DISTRIBUTION

Example 4.6

The random variable $X \sim \mathrm{B}(n, p)$. Given that $\mathrm{E}(X) = 12$ and $\mathrm{Var}(X) = 7.5$, find:

a the value of n and of p

b $\mathrm{P}(X = 11)$.

Example 4.7

X is $B(\ n, p)$ with mean 5 and standard deviation 2. Find the values of n and p.

Example 4.8

In Restaurant Bijoux 13% of customers rated the food as ‘poor’, 22% of customers rated the food as ‘satisfactory’ and 65% rated it as ‘good’. A random sample of 12 customers who went for a meal at Restaurant Bijoux was taken.

i Find the probability that more than 2 and fewer than 12 of them rated the food as ‘good’. [3]

On a separate occasion, a random sample of n customers who went for a meal at the restaurant was taken.

ii Find the smallest value of n for which the probability that at least 1 person will rate the food as ‘poor’ is greater than 0.95. [3]

Cambridge International AS & A Level Mathematics 9709 Paper 62 Q3 June 2012

Example 4.9

The mean number of defective batteries in packs of 20 is 1.6. Use a binomial distribution to calculate the probability that a randomly chosen pack of 20 will have more than 2 defective batteries.

[Cambridge International AS and A Level Mathematics 9709, Paper 61 Q1 November 2009]

THE MODE OF BINOMIAL DISTRIBUTION

The mode is the value of X that is most likely to occur.
From the probability distribution sketches above, it can be seen that

- When p= 0.5 and is odd, there are two modes,
- Otherwise the distribution has one mode.

The mode can be found by calculating all the probabilities and find value of X with the highest probability of values of X close to the mean np.

Example 4.6

The probability that a student is awarded a distinction in the Mathematics examination is 0.05.
In a randomly selected group of students, what is the most likely number of students awarded a distinction?

HOMEWORK:BINOMIAL DISTRIBUTION– VARIANT 62

1 A shop sells old video tapes, of which 1 in 5 on average are known to be damaged.

(i) A random sample of 15 tapes is taken. Find the probability that at most 2 are damaged. [3]

(ii) Find the smallest value of n if there is a probability of at least 0.85 that a random sample of n tapes contains at least one damaged tape. [3]

Answers: (i) 0.398; (ii) 9; (iii) 0.972.

J04/Q7

2 A survey of adults in a certain large town found that 76% of people wore a watch on their left wrist, 15% wore a watch on their right wrist and 9% did not wear a watch.

(i) A random sample of 14 adults was taken. Find the probability that more than 2 adults did not wear a watch. [4]

Answers: (i) 0.126; (ii) 0.281. **J06/Q7**

3 The probability that New Year's Day is on a Saturday in a randomly chosen year is $\frac{1}{7}$.

(i) 15 years are chosen randomly. Find the probability that at least 3 of these years have New Year's Day on a Saturday. [4]

Answers: (i) 0.365; (ii) 0.576. **J07/Q6**

4 A die is biased so that the probability of throwing a 5 is 0.75 and the probabilities of throwing a 1, 2, 3, 4 or 6 are all equal.

(i) The die is thrown three times. Find the probability that the result is a 1 followed by a 5 followed by any even number. [3]

(ii) Find the probability that, out of 10 throws of this die, at least 8 throws result in a 5. [3]

Answers: (i) 0.00563; (ii) 0.526; (iii) 0.956. **J08/Q7**

5 On a certain road 20% of the vehicles are trucks, 16% are buses and the remainder are cars.

(i) A random sample of 11 vehicles is taken. Find the probability that fewer than 3 are buses. [3]

Answers: (i) 0.748; (ii) 0.887. **J09/Q3**

6 (i) A manufacturer of biscuits produces 3 times as many cream ones as chocolate ones. Biscuits are chosen randomly and packed into boxes of 10. Find the probability that a box contains equal numbers of cream biscuits and chocolate biscuits. [2]

(ii) A random sample of 8 boxes is taken. Find the probability that exactly 1 of them contains equal numbers of cream biscuits and chocolate biscuits. [2]

Answers: (i) 0.0584; (ii) 0.307; (iii) 0.829. **N02/Q6**

7 **(i)** State two conditions which must be satisfied for a situation to be modelled by a binomial distribution. [2]

In a certain village 28% of all cars are made by Ford.

(ii) 14 cars are chosen randomly in this village. Find the probability that fewer than 4 of these cars are made by Ford. [4]

Answers: (ii) 0.419; (iii) 0.0782.

N04/Q7

8 A manufacturer makes two sizes of elastic bands: large and small. 40% of the bands produced are large bands and 60% are small bands. Assuming that each pack of these elastic bands contains a random selection, calculate the probability that, in a pack containing 20 bands, there are

(i) equal numbers of large and small bands, [2]

(ii) more than 17 small bands. [3]

Answers: (i) 0.117; (ii) 0.00361; (iii) 0.556.

N06/Q7

9 On any occasion when a particular gymnast performs a certain routine, the probability that she will perform it correctly is 0.65, independently of all other occasions.

(i) Find the probability that she will perform the routine correctly on exactly 5 occasions out of 7. [2]

Answers: (i) 0.298; (ii) 0.118; (iii) 13.

N07/Q6

10 Two unbiased tetrahedral dice each have four faces numbered 1, 2, 3 and 4. The two dice are thrown together and the sum of the numbers on the faces on which they land is noted. Find the expected number of occasions on which this sum is 7 or more when the dice are thrown together 200 times. [4]

Answer: 37.5.

N09/62/Q2

11 **(i)** State three conditions that must be satisfied for a situation to be modelled by a binomial distribution. [2]

On any day, there is a probability of 0.3 that Julie's train is late.

(ii) Nine days are chosen at random. Find the probability that Julie's train is late on more than 7 days or fewer than 2 days. [3]

Answers: (ii) 0.196; (iii) 0.480.

N10/62/Q6

12 A biased die was thrown 20 times and the number of 5s was noted. This experiment was repeated many times and the average number of 5s was found to be 4.8. Find the probability that in the next 20 throws the number of 5s will be less than three. [4]

Answer: 0.109.

J11/62/Q1

13 In Restaurant Bijoux 13% of customers rated the food as ‘poor’, 22% of customers rated the food as ‘satisfactory’ and 65% rated it as ‘good’. A random sample of 12 customers who went for a meal at Restaurant Bijoux was taken.

(i) Find the probability that more than 2 and fewer than 12 of them rated the food as ‘good’. [3]

On a separate occasion, a random sample of n customers who went for a meal at the restaurant was taken.

(ii) Find the smallest value of n for which the probability that at least 1 person will rate the food as ‘poor’ is greater than 0.95. [3]

Answers: (i) 0.993 (ii) 22 **J12/62/Q3**

14 Robert uses his calculator to generate 5 random integers between 1 and 9 inclusive.

(i) Find the probability that at least 2 of the 5 integers are less than or equal to 4. [3]

Robert now generates n random integers between 1 and 9 inclusive. The random variable X is the number of these n integers which are less than or equal to a certain integer k between 1 and 9 inclusive. It is given that the mean of X is 96 and the variance of X is 32.

(ii) Find the values of n and k. [4]

Answer: 0.735 *Answers:* 144, 6 **J13/62/Q4**

15 In a certain country 12% of houses have solar heating. 19 houses are chosen at random. Find the probability that fewer than 4 houses have solar heating. [4]

Answer: 0.813 **J14/62/Q1**

16 A fair die is thrown 10 times. Find the probability that the number of sixes obtained is between 3 and 5 inclusive. [3]

Answer: 0.222 **J15/62/Q1**

17 There are a large number of students in Luttley College. 60% of the students are boys. Students can choose exactly one of Games, Drama or Music on Friday afternoons. It is found that 75% of the boys choose Games, 10% of the boys choose Drama and the remainder of the boys choose Music. Of the girls, 30% choose Games, 55% choose Drama and the remainder choose Music.

(i) 6 boys are chosen at random. Find the probability that fewer than 3 of them choose Music. [3]

(ii) 5 Drama students are chosen at random. Find the probability that at least 1 of them is a boy. [6]

Answers: (i) 0.953; (ii) 0.701 **N11/62/Q6**

18 **(i)** Four fair six-sided dice, each with faces marked 1, 2, 3, 4, 5, 6, are thrown. Find the probability that the numbers shown on the four dice add up to 5. [3]

(ii) Four fair six-sided dice, each with faces marked 1, 2, 3, 4, 5, 6, are thrown on 7 occasions. Find the probability that the numbers shown on the four dice add up to 5 on exactly 1 or 2 of the 7 occasions. [4]

N14/62/Q3

Answers: $\frac{1}{324}$, 0.0124

HOMEWORK:BINOMIAL DISTRIBUTION VARIANT 61 AND 63

1 In a certain town, 76% of cars are fitted with satellite navigation equipment. A random sample of 11 cars from this town is chosen. Find the probability that fewer than 10 of these cars are fitted with this equipment. [4]

Answer: 0.781 **61/N15/1**

2 In a certain country, 68% of households have a printer. Find the probability that, in a random sample of 8 households, 5, 6 or 7 households have a printer. [4]

Answer i) 0.722 **61/J15/6**

A factory makes water pistols, 8% of which do not work properly.

(i) A random sample of 19 water pistols is taken. Find the probability that at most 2 do not work properly. [3]

(ii) In a random sample of n water pistols, the probability that at least one does not work properly is greater than 0.9. Find the smallest possible value of n. [3]

Answer: 0.809, 28 **63/N15/7**

3 The number of books read by members of a book club each year has the binomial distribution B(12, 0.7).

(i) State the greatest number of books that could be read by a member of the book club in a particular year and find the probability that a member reads this number of books. [2]

(ii) Find the probability that a member reads fewer than 10 books in a particular year. [3]

Answer: i) 12, 0.0138 ii) 0.747 **63/N14/3**

4 (i) Four fair six-sided dice, each with faces marked 1, 2, 3, 4, 5, 6, are thrown. Find the probability that the numbers shown on the four dice add up to 5. [3]

(ii) Four fair six-sided dice, each with faces marked 1, 2, 3, 4, 5, 6, are thrown on 7 occasions. Find the probability that the numbers shown on the four dice add up to 5 on exactly 1 or 2 of the 7 occasions. [4]

Answer: i) 0.00309 ii) 0.0214 **62/N14/3**

5 In Marumbo, three quarters of the adults own a cell phone.

(i) A random sample of 8 adults from Marumbo is taken. Find the probability that the number of adults who own a cell phone is between 4 and 6 inclusive. [3]

Answer: i) 0.606 **62/N14/7**

6* Screws are sold in packets of 15. Faulty screws occur randomly. A large number of packets are tested for faulty screws and the mean number of faulty screws per packet is found to be 1.2.

(i) Show that the variance of the number of faulty screws in a packet is 1.104. [2]

(ii) Find the probability that a packet contains at most 2 faulty screws. [3]

Damien buys 8 packets of screws at random.

(iii) Find the probability that there are exactly 7 packets in which there is at least 1 faulty screw. [4]

Answer ii) 0.887 iii) 0.216 **61/N14/5**

---

7 (i) State three conditions which must be satisfied for a situation to be modelled by a binomial distribution. [2]

George wants to invest some of his monthly salary. He invests a certain amount of this every month for 18 months. For each month there is a probability of 0.25 that he will buy shares in a large company, there is a probability of 0.15 that he will buy shares in a small company and there is a probability of 0.6 that he will invest in a savings account.

(ii) Find the probability that George will buy shares in a small company in at least 3 of these 18 months. [3]

Answer: constant p, given n, only two outcomes ii) 0.520 **61/J14/3**

---

8 In a large consignment of mangoes, 15% of mangoes are classified as small, 70% as medium and 15% as large.

(i) Yue-chen picks 14 mangoes at random. Find the probability that fewer than 12 of them are medium or large. [3]

(ii) Yue-chen picks n mangoes at random. The probability that none of these n mangoes is small is at least 0.1. Find the largest possible value of n. [3]

Answer: i) 0.352 ii) 14 **63/N13/3**

9 The 12 houses on one side of a street are numbered with even numbers starting at 2 and going up to 24. A free newspaper is delivered on Monday to 3 different houses chosen at random from these 12. Find the probability that at least 2 of these newspapers are delivered to houses with numbers greater than 14. [4]

Answer: 0.364 **63/J13/2**

---

10 In a certain country, on average one student in five has blue eyes.

(i) For a random selection of n students, the probability that none of the students has blue eyes is less than 0.001. Find the least possible value of n. [3]

Answer: 31 **63/J13/4**

---

11 Fiona uses her calculator to produce 12 random integers between 7 and 21 inclusive. The random variable X is the number of these 12 integers which are multiples of 5.

(i) State the distribution of X and give its parameters. [3]

(ii) Calculate the probability that X is between 3 and 5 inclusive. [3]

Fiona now produces n random integers between 7 and 21 inclusive.

(iii) Find the least possible value of n if the probability that none of these integers is a multiple of 5 is less than 0.01. [3]

Answer: i) 12, 0.2 ii) 0.422 iii) 21 **61/J13/5**

---

12 A company set up a display consisting of 20 fireworks. For each firework, the probability that it fails to work is 0.05, independently of other fireworks.

(i) Find the probability that more than 1 firework fails to work. [3]

Answer: i) 0.264 **61/N12/5**

---

13 In a certain mountainous region in winter, the probability of more than 20 cm of snow falling on any particular day is 0.21.

(i) Find the probability that, in any 7-day period in winter, fewer than 5 days have more than 20 cm of snow falling. [3]

(ii) For 4 randomly chosen 7-day periods in winter, find the probability that exactly 3 of these periods will have at least 1 day with more than 20 cm of snow falling. [4]

Answer: i) 0.994 ii) 0.405 **61/J12/4**

14 Human blood groups are identified by two parts. The first part is A, B, AB or O and the second part (the Rhesus part) is + or –. In the UK, 35% of the population are group A+, 8% are B+, 3% are AB+, 37% are O+, 7% are A–, 2% are B–, 1% are AB– and 7% are O–.

(i) A random sample of 9 people in the UK who are Rhesus + is taken. Find the probability that fewer than 3 are group O+. [6]

Answer: i) 0.156 ii) 0.0854 **63/N11/6**

---

15 Biscuits are sold in packets of 18. There is a constant probability that any biscuit is broken, independently of other biscuits. The mean number of broken biscuits in a packet has been found to be 2.7. Find the probability that a packet contains between 2 and 4 (inclusive) broken biscuits. [4]

Answer: 0.655 **61/J11/1**

---

16 (a) (i) Find the probability of getting at least one 3 when 9 fair dice are thrown. [2]

(ii) When n fair dice are thrown, the probability of getting at least one 3 is greater than 0.9. Find the smallest possible value of n. [4]

Answer: i) 0.806 ii) 13 **61/J11/7**

---

17 Christa takes her dog for a walk every day. The probability that they go to the park on any day is 0.6. If they go to the park there is a probability of 0.35 that the dog will bark. If they do not go to the park there is a probability of 0.75 that the dog will bark.

(i) Find the probability that they go to the park on more than 5 of the next 7 days. [2]

(ii) Find the probability that the dog barks on any particular day. [2]

(iii) Find the variance of the number of times they go to the park in 30 days. [1]

Answer: i) 0.159 ii) 0.51 iii) 7.2 **63/N10/3**

---

18 In the holidays Martin spends 25% of the day playing computer games. Martin's friend phones him once a day at a randomly chosen time.

(i) Find the probability that, in one holiday period of 8 days, there are exactly 2 days on which Martin is playing computer games when his friend phones. [2]

Answer: 0.311 **61/J10/5**

19 Passengers are travelling to Picton by minibus. The probability that each passenger carries a backpack is 0.65, independently of other passengers. Each minibus has seats for 12 passengers.

(i) Find the probability that, in a full minibus travelling to Picton, between 8 passengers and 10 passengers inclusive carry a backpack. [3]

(ii) Passengers get on to an empty minibus. Find the probability that the fourth passenger who gets on to the minibus will be the first to be carrying a backpack. [2]

Answer: (i) 0.541 (ii) 0.0279 (iii) 0.290

J16/63/Q7

20 Eggs are sold in boxes of 20. Cracked eggs occur independently and the mean number of cracked eggs in a box is 1.4.

(i) Calculate the probability that a randomly chosen box contains exactly 2 cracked eggs. [3]

(ii) Calculate the probability that a randomly chosen box contains at least 1 cracked egg. [2]

(iii) A shop sells n of these boxes of eggs. Find the smallest value of n such that the probability of there being at least 1 cracked egg in each box sold is less than 0.01. [2]

Answer: (i) 0.252 (ii) 0.766 (iii) 18

J17/61/Q5

21 Hebe attempts a crossword puzzle every day. The number of puzzles she completes in a week (7 days) is denoted by X.

(i) State two conditions that are required for X to have a binomial distribution. [2]

On average, Hebe completes 7 out of 10 of these puzzles.

(ii) Use a binomial distribution to find the probability that Hebe completes at least 5 puzzles in a week. [3]

(iii) Use a binomial distribution to find the probability that, over the next 10 weeks, Hebe completes 4 or fewer puzzles in exactly 3 of the 10 weeks. [3]

Answers: (i) 1 Constant probability (of completing) 2 Independent trials/events (ii) 0.647
(iii) 0.251

J17/63/Q5

22 Visitors to a Wildlife Park in Africa have independent probabilities of 0.9 of seeing giraffes, 0.95 of seeing elephants, 0.85 of seeing zebras and 0.1 of seeing lions.

(i) Find the probability that a visitor to the Wildlife Park sees all these animals. [1]

(ii) Find the probability that, out of 12 randomly chosen visitors, fewer than 3 see lions. [3]

(iii) 50 people independently visit the Wildlife Park. Find the mean and variance of the number of these people who see zebras. [2]

Answers (i) 0.0727 (ii) 0.889 (iii) 42.5, 6.375 **N16/61/Q3**

---

23 An experiment consists of throwing a biased die 30 times and noting the number of 4s obtained. This experiment was repeated many times and the average number of 4s obtained in 30 throws was found to be 6.21.

(i) Estimate the probability of throwing a 4. [1]

Hence

(ii) find the variance of the number of 4s obtained in 30 throws, [1]

(iii) find the probability that in 15 throws the number of 4s obtained is 2 or more. [3]

Answers: (i) 0.207 (ii) 4.92 (iii) 0.848 or 0.8485 **N17/61/Q3**

THE GEOMETRIC DISTRIBUTION

Some children's games include having to throw a 6 on your turn to be able to start moving on the board. It can be hugely frustrating when you have to wait while others are already moving. But just what is the mathematics of this situation?

The likelihood of only needing one throw is $\frac{1}{6}$, but what happens after that?

If I get started on the second throw I know I did not get a 6 the first throw and then I did get a 6 the next throw, so the probability is $\frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$.

If I get started on the third go, I had to not get a 6 twice and then get a 6, so the probability is $\left(\frac{5}{6}\right)^2 \times \frac{1}{6}$.

If I start on the tenth go I must have not got a 6 on all of the first nine throws and then got a 6, so the probability is $\left(\frac{5}{6}\right)^9 \times \frac{1}{6}$. I can see a really strong pattern that I know will always be the case: if I start on the rth go I must have not got a 6 on all of the first $(r-1)$ throws and then got a 6, and the probability will be $\left(\frac{5}{6}\right)^{r-1} \times \frac{1}{6}$.

A discrete random variable, X, is said to have a geometric distribution, and is defined by its parameter p, if it meets the following criteria.

- The repeated trials are independent.
- The repeated trials can be infinite in number.
- There are just two possible outcomes for each trial (i.e. success or failure).
- The probability of success in each trial, p, is constant.

The parameter of the geometric distribution is p, the probability of a 'success' on any one trial.

For simplicity the distribution is often written as $X \sim \text{Geo}(p)$.

The geometric distribution is defined as

$$\mathrm{P}(X = r) = q^{r-1}p \qquad r = 1, 2, 3, \ldots, \qquad q = 1 - p.$$

When $X \sim \text{Geo}(p)$ and $q = 1 - p$, then

- $P(X \leqslant r) = 1 - q^r$
- $P(X > r) = q^r$

NOTE:

- X cannot take the value 0,
- the number of trials could be infinite, although this is unlikely in practice!

THE MODE OF THE GEOMETRIC DISTRIBUTION

From the diagrams, you can see that the mode of any geometric distribution is 1. This means that for any value of p, one attempt is the most likely number of attempts to obtain the first success. This is quite a surprising result.

Example 5.1

If $X \sim \text{Geo}(0.4)$ find the probability that $X = 3$.

Example 5.2

If $X \sim \text{Geo}(0.8)$ find $P(X \leq 5)$ correct to 4 significant figures.

Example 5.3

If $X \sim \text{Geo}(0.4)$ find

a) $P(X = 2)$ **b)** $P(X > 4)$ **c)** $P(X = 6)$ **d)** $P(X = 6 | X > 4)$.

Example 5.4

Jack is playing a board game in which he needs to throws a six with an ordinary die in order to start the game. Find the probability that

(a) exactly four attempts are needed to obtain a six,
(b) at least two attempts are needed,
(c) he is successful in throwing a six in three or fewer attempts,
(d) he needs more than three attempts to obtain a six.

Example 5.5

In a particular country, 18% of adults wear contact lenses. Adults are randomly selected and interviewed one at a time. Find the probability that the first adult who wears contact lenses is:

a one of the first 15 interviewed

b not one of the first nine interviewed.

Example 5.6

On a particular production line the probability that an item is faulty is 0.08. In a quality control test, items are selected at random from the production line. It is assumed that quality of an item is independent of that of other items.

(a) Find the probability that the first faulty item

  (i) does not occur in the first six selected,
  (ii) occurs in fewer than five selections.

(b) There is to be at least a 90% chance of picking a faulty item on or before the nth attempt. What is the smallest number n?

THE MEAN OF A GEOMETRIC DISTRBUTION

If $X \sim \text{Geo}(p)$ then $\text{E}(X) = \frac{1}{p}$.

Example 5.7

If $X \sim \text{Geo}(0.8)$, find

a) $\text{E}(X)$
b) $\text{P}\{X < \text{E}(X)\}$.

Example 5.8

If $X \sim \text{Geo}(p)$, and $\text{E}(X) = 4$, find

a) the value of p
b) $\text{P}\{X > \text{E}(X)\}$
c) the least integer n such that $\text{P}(X > n) < 1\%$.

Example 5.9

Carmen wants to take a taxi. Where she is, 5% of the vehicles are taxis. X is the number of vehicles that she sees up to and including when she sees a taxi.

a) On average, how many vehicles will she see up to and including when she sees a taxi?

b) Calculate the probability she will see a taxi in the first five vehicles.

c) Calculate the probability that she is still waiting to see a taxi after she has seen 30 vehicles.

d) **Evaluate** the mode of X, i.e. what number vehicle is the most likely to be the first taxi she sees?

MODELLING WITH THE GEOMETRIC DISTRIBUTION

The geometric distribution is based on the waiting time until the first 'success' is registered in a sequence of Bernoulli trials. In Chapter 7 you met the **binomial distribution** which counted the number of successes in a fixed number of Bernoulli trials, so there are strong similarities in the two contexts and you should look back at Section 7.3 to remind yourself of the sorts of contexts in which the assumptions of **independence** and constant probability may not hold.

Example 5.10

Anya makes repeated attempts to thread a needle. The number of attempts up to and including the first success is denoted by X.

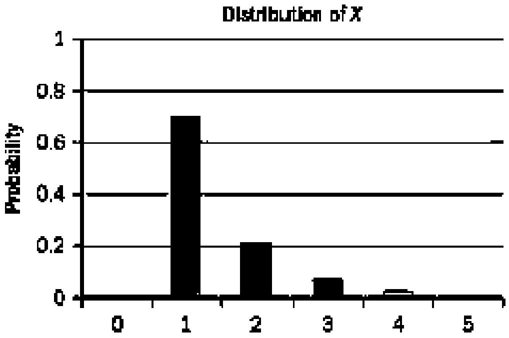
a) State two conditions necessary for X to have a geometric distribution.

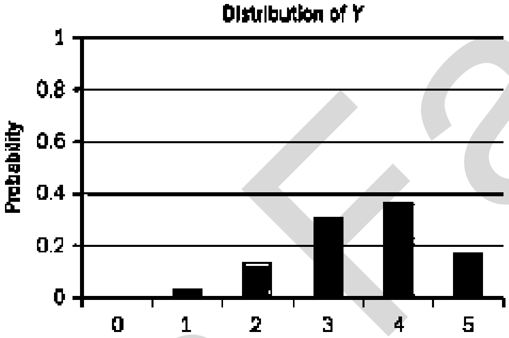
b) Assuming that X has the distribution Geo(0.4) find the probability that she needs more than three attempts before she threads the needle successfully.

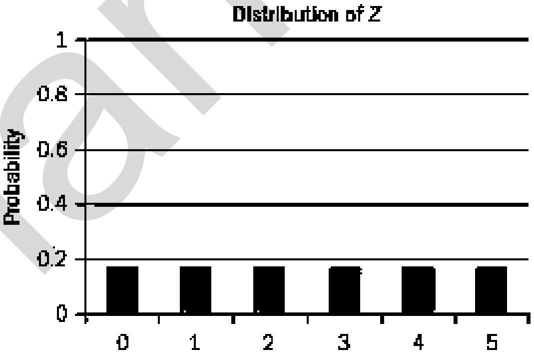
c) Suggest a reason why one of the conditions you gave in part (a) might not be satisfied in this context.

Example 5.11

The diagrams illustrate all or part of the distributions of the discrete random variables *X*, *Y*, *Z*.


Distribution of *X*


Distribution of *Y*


Distribution of *Z*

One of these variables has the distribution Geo(0.7). State, with a reason, which variable it is.

6. If X is a geometric random variable and $\mathrm{E}(X) = 4$, find

a) $\mathrm{P}(X = 1)$

b) $\mathrm{P}(X > 3)$.

7. A fair spinner has eight equal sections numbered 1 to 8. It is to be spun until it lands on a prime number. What is the probability that it takes more than four spins to see a prime for the first time?

8. Only 2% of vehicles on a particular stretch of road are prepared to stop to give a lift to hitch-hikers. Amir wants to hitch-hike on that road. Find the probability that he has not got a lift yet after 20 vehicles have gone past him.

Do you think the geometric distribution is a good model for this situation? Explain, giving a reason for your answer.

EXAM STYLE QUESTIONS

9. Thirty per cent of the members of a very large group of fitness clubs are over 40 years old. The company owning the clubs want to interview a random sample of members who are over 40. The secretary uses the database of all members, and selects members at random until she finds ones who are over 40.

If X denotes the number of members selected up to and including the first member who is over 40, calculate:

a) $\mathrm{P}(X = 4)$ [3]

b) $\mathrm{P}(X > 4)$ [2]

c) $\mathrm{P}(X < 6)$. [3]

10. Adeline has to light a gas fire for her grandmother every day. It is hard to light, and repeated attempts have a probability of $\frac{1}{3}$ of success, independent of one another.

a) If X is the number of attempts needed until the fire lights on a particular day, state the distribution of X. [1]

b) Using the distribution you stated in part (a), calculate the probabilities

i) $\mathrm{P}(X = 4)$ **ii)** $\mathrm{P}(X < 3)$. [4]

c) State $\mathrm{E}(X)$. [1]

d) Calculate the probability that the first date in March on which Adeline needs fewer than the mean number of attempts to light the fire is March 6th. [3]

11. a) If X_1, X_2 are both geometric random variables with parameter p, show that $\mathrm{P}(X_1 + X_2 = 3) = 2p^2q$. [3]

b) If Y_1, Y_2 are both geometric random variables with parameter 0.3, find $\mathrm{P}(Y_1 + Y_2 > 3)$. [2]

12. $Y \sim \mathrm{Geo}(p)$.

a) Show that $P(Y \text{ is even}) = qp + q^3p + q^5p + \ldots$ (where $q = 1-p$). [3]

b) Use the formula for the sum to infinity of a geometric progression to show that $\mathrm{P}(Y \text{ is even}) = \frac{q}{1+q}$. [3]

13. Christina and Novak play a game where they take turns to throw a fair dice. Christina goes first and the game ends when either player throws the first 6. Calculate:

a) P(Novak wins on his first throw)

b) P(Christina wins on her third throw)

c) P(Novak gets a third throw)

d) P(Christina throws exactly three times)

e) P(Novak wins the game).

Summary exercise 8 page 143

1. **a)** 0.1024 **b)** 0.262

2. 1.11

3. **a)** $\frac{4}{27}$ **b)** 3 **c)** $\frac{8}{27}$

4. **a)** 0.144 **b)** 0.216

c) $P(X=6 \mid X>3) = \frac{P(X=6)}{P(X>3)} = \frac{0.6^5 \times 0.4}{0.6^3} = 0.6^2 \times 0.4$
$= 0.144$

d) a memoryless property

5. 0.613

6. **a)** $\frac{1}{4}$ **b)** $\frac{27}{64}$

7. 0.0625

8. 0.668

9. **a)** 0.1029 **b)** 0.2401 **c)** 0.832

10. **a)** $X \sim \text{Geo}\left(\frac{1}{3}\right)$ **b)** **i)** 0.0988 **ii)** $\frac{5}{9}$

c) 3 **d)** 0.00963

11. **b)** $P(Y_1 + Y_2 > 3) = 1 - P(Y_1 + Y_2 = 2, 3) =$
$1 - (0.3^2 + 2 \times 0.3^2 \times 0.7) = 0.784$

13. **a)** $\frac{5}{36}$

b) 0.0804 (3 s.f.)

c) 0.402 (3 s.f.)

d) 0.147

e) $\frac{5}{11}$

GEOMETRIC DISTRIBUTION PAST PAPERS

1 On average 1 in 20 members of the population of this country has a particular DNA feature. Members of the population are selected at random until one is found who has this feature.

(i) Find the probability that the first person to have this feature is

(a) the sixth person selected, [3]

(b) not among the first 10 people selected. [3]

(ii) Find the expected number of people selected. [2]

Answer: i) a) 0.0387 b) 0.599 ii) 20

---

2 The probability that a certain sample of radioactive material emits an alpha-particle in one unit of time is 0.14. In one unit of time no more than one alpha-particle can be emitted. The number of units of time up to and including the first in which an alpha-particle is emitted is denoted by T.

(i) Find the value of

(a) $P(T = 5)$, [3]

(b) $P(T < 8)$. [3]

(ii) State the value of $E(T)$. [2]

Answer: i) a) 0.0766 b) 0.652 ii) 50/7

---

3 Henry makes repeated attempts to light his gas fire. He makes the modelling assumption that the probability that the fire will light on any attempt is $\frac{1}{3}$.

Let X be the number of attempts at lighting the fire, up to and including the successful attempt.

(i) Name the distribution of X, stating a further modelling assumption needed. [2]

In the rest of this question, you should use the distribution named in part **(i)**.

(ii) Calculate

(a) $P(X = 4)$, [3]

(b) $P(X < 4)$. [3]

(iii) State the value of $E(X)$. [1]

(iv) Henry has to light the fire once a day, starting on March 1st. Calculate the probability that the first day on which fewer than 4 attempts are needed to light the fire is March 3rd. [3]

Answer: i) Geometric, Each attempt is independent ii) a) 8/81 b) 19/27 iii) 3 iv) 0.0618

4 A coin is biased so that the probability that it will show heads on any throw is $\frac{2}{3}$. The coin is thrown repeatedly.

The number of throws up to and including the first head is denoted by X. Find

(i) $\mathrm{P}(X = 4)$, [3]

(ii) $\mathrm{P}(X < 4)$, [3]

(iii) $\mathrm{E}(X)$. [2]

Answer: i) 2/81 ii) 26/27 iii) 3/2

5 **(i)** A random variable X has the distribution $\mathrm{Geo}\left(\frac{1}{5}\right)$. Find

(a) $\mathrm{E}(X)$, [2]

(b) $\mathrm{P}(X = 4)$, [2]

(c) $\mathrm{P}(X > 4)$. [2]

(ii) A random variable Y has the distribution $\mathrm{Geo}(p)$, and $q = 1 - p$.

(a) Show that $\mathrm{P}(Y \text{ is odd}) = p + q^2p + q^4p + \dots$. [1]

(b) Use the formula for the sum to infinity of a geometric progression to show that

$$\mathrm{P}(Y \text{ is odd}) = \frac{1}{1+q}.$$ [4]

Answer: i) a) 5 b) 64/625 c) 256/625

6 Erika is a birdwatcher. The probability that she will see a woodpecker on any given day is $\frac{1}{8}$. It is assumed that this probability is unaffected by whether she has seen a woodpecker on any other day.

(i) Calculate the probability that Erika first sees a woodpecker

(a) on the third day, **[3]**

(b) after the third day. **[3]**

(ii) Find the expectation of the number of days up to and including the first day on which she sees a woodpecker. **[1]**

(iii) Calculate the probability that she sees a woodpecker on exactly 2 days in the first 15 days. **[3]**

Answer: i) a) 49/512 b) 343/512 ii) 8 iii) 0.289

7 30% of people own a Talk-2 phone. People are selected at random, one at a time, and asked whether they own a Talk-2 phone. The number of people questioned, up to and including the first person who owns a Talk-2 phone, is denoted by X. Find

(i) $\mathrm{P}(X = 4)$, **[3]**

(ii) $\mathrm{P}(X > 4)$, **[2]**

(iii) $\mathrm{P}(X < 6)$. **[3]**

Answer: i) 0.103 ii) 0.240 iii) 0.832

8 Andy makes repeated attempts to thread a needle. The number of attempts up to and including his first success is denoted by X.

(i) State two conditions necessary for X to have a geometric distribution. **[2]**

(ii) Assuming that X has the distribution Geo(0.3), find

(a) $P(X = 5)$, **[2]**

(b) $P(X > 5)$. **[3]**

(iii) Suggest a reason why one of the conditions you have given in part **(i)** might not be satisfied in this context. **[2]**

Answer: i) attempts at threading are independent, prop of threading constant ii) a) 0.0720 b) 0.168 iii) likely to improve with practice hence independent unlikely

---

9 R and S are independent random variables each having the distribution Geo(p).

(i) Find $P(R = 1 \text{ and } S = 1)$ in terms of p. **[1]**

(ii) Show that $P(R = 3 \text{ and } S = 3) = p^2q^4$, where $q = 1 - p$. **[1]**

(iii) Use the formula for the sum to infinity of a geometric series to show that

$$P(R = S) = \frac{p}{2-p}.$$ **[5]**

Answer: i) p^2 ii) $(q^2p)^2$

---

10 The proportion of people who watch *West Street* on television is 30%. A market researcher interviews people at random in order to contact viewers of *West Street*. Each day she has to contact a certain number of viewers of *West Street*.

(i) Near the end of one day she finds that she needs to contact just one more viewer of *West Street*. Find the probability that the number of further interviews required is

(a) 4, **[3]**

(b) less than 4. **[3]**

(ii) Near the end of another day she finds that she needs to contact just two more viewers of *West Street*. Find the probability that the number of further interviews required is

(a) 5, **[4]**

(b) more than 5. **[2]**

Answer: i) a) 0.103 b) 0.657 ii) a) 0.123 b) 0.528

11 The random variable X has the distribution Geo(0.2). Find

(i) $P(X = 3)$, **[2]**

(ii) $P(3 \leqslant X \leqslant 5)$, **[3]**

(iii) $P(X > 4)$. **[3]**

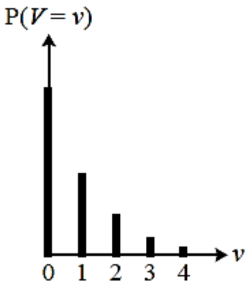
Two independent values of X are found.

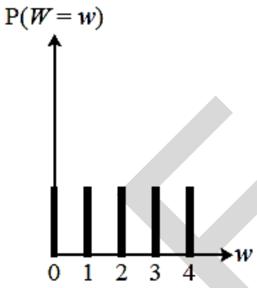
(iv) Find the probability that the total of these two values is 3. **[3]**

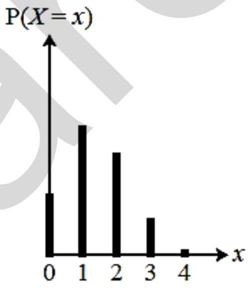
Answer: i) 0.128 ii) 0.312 iii) 0.4096 iv) 0.064

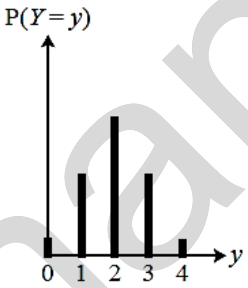
---

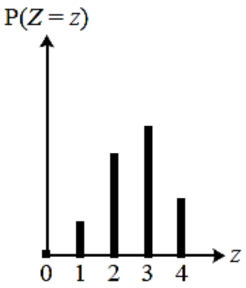
12 The diagrams illustrate all or part of the probability distributions of the discrete random variables V, W, X, Y and Z.











(i) One of these variables has the distribution Geo($\frac{1}{2}$). State, with a reason, which variable this is. **[2]**

(ii) One of these variables has the distribution B(4, $\frac{1}{2}$). State, with reasons, which variable this is. **[3]**

Answer: i) V ii) Y

13 60% of the voters at a certain polling station are women. Voters enter the polling station one at a time. The number of voters who enter, up to and including the first woman, is denoted by X.

(i) State a suitable distribution that can be used as a model for X, giving the value(s) of any parameter(s). State also any necessary condition(s) for this distribution to be a good model. **[4]**

Use the distribution stated in part **(i)** to find

(ii) $P(X = 4)$, **[2]**

(iii) $P(X \geqslant 4)$. **[2]**

Answer: i) Geo (0.6) , Each voter has same probability ii) 0.0384 iii) 0.064

---

14 **(i)** A clock is designed to chime once each hour, on the hour. The clock has a fault so that each time it is supposed to chime there is a constant probability of $\frac{1}{10}$ that it will not chime. It may be assumed that the clock never stops and that faults occur independently. The clock is started at 5 minutes past midnight on a certain day. Find the probability that the first time it does not chime is

(a) at 0600 on that day, **[3]**

(b) before 0600 on that day. **[3]**

(ii) Another clock is designed to chime twice each hour: on the hour and at 30 minutes past the hour. This clock has a fault so that each time it is supposed to chime there is a constant probability of $\frac{1}{20}$ that it will not chime. It may be assumed that the clock never stops and that faults occur independently. The clock is started at 5 minutes past midnight on a certain day.

(a) Find the probability that the first time it does not chime is at either 0030 or 0130 on that day. **[2]**

(b) Use the formula for the sum to infinity of a geometric progression to find the probability that the first time it does not chime is at 30 minutes past some hour. **[3]**

Answer: i) a) 0.059 b) 0.4095 ii) a) 0.0951 b) 0.513

15 Sandra makes repeated, independent attempts to hit a target. On each attempt, the probability that she succeeds is 0.1.

(i) Find the probability that

(a) the first time she succeeds is on her 5th attempt, **[2]**

(b) the first time she succeeds is after her 5th attempt, **[2]**

(c) the second time she succeeds is before her 4th attempt. **[4]**

Jill also makes repeated attempts to hit the target. Each attempt of either Jill or Sandra is independent. Each time that Jill attempts to hit the target, the probability that she succeeds is 0.2. Sandra and Jill take turns attempting to hit the target, with Sandra going first.

(ii) Find the probability that the first person to hit the target is Sandra, on her

(a) 2nd attempt, **[2]**

(b) 10th attempt. **[3]**

Answer: i) a) 0.0656 b) 0.59 c) 0.028 ii) a) 0.072 b) 0.0052

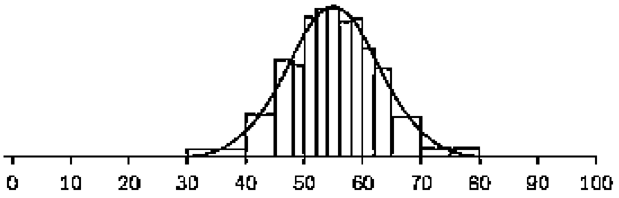
THE NORMAL DISTRIBUTION

The normal distribution is a type of **continuous probability distribution.** This means that:

- It relates to a continuous variable (height, weight etc.).
- It describes the **probability** of this variable taking a particular range of values.

Did you know?
Physicists sometimes refer to this as the Gaussian distribution.

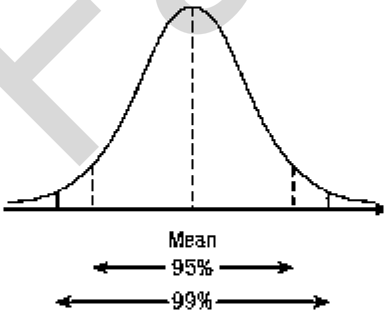
Generally, the normal distribution looks like this:



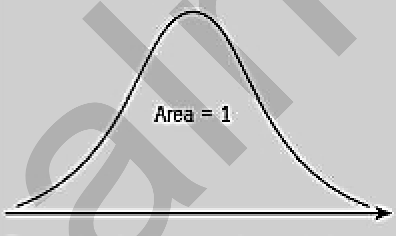
The form of the function is $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$. This formula is not part of the Cambridge syllabus and so will not be required.

The idealised normal distribution has the following properties:

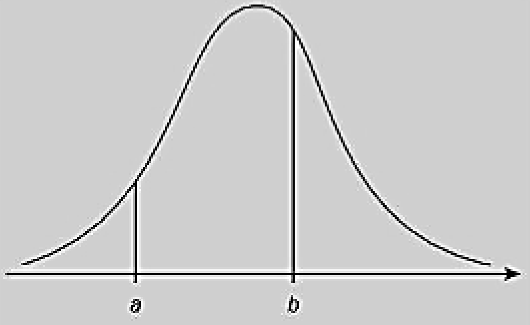
- It is symmetric.
- It is infinite in both directions.
- It has a single peak at the centre.
- It is continuous.
- 95% of values lie within approximately 2 **standard deviations** of the **mean.**
- 99% lie within approximately 3 standard deviations of the mean.



The following will enable us to calculate probabilities:



The total under the normal curve is 1.



The probability that X lies between a and b is the area under the curve between $x=a$ and $x=b$.

STANDARDIZED SCORES

The normal distribution allows us to make comparisons between individuals in a particular **normal population**. However, it becomes harder with different normal populations – for example, we use different criteria to judge a 'tall man' than we use for a 'tall woman'.

One way to compare different normal populations is to standardise the scores – by looking at their distance from the mean, then dividing by the size of the standard deviation.

- To find the **standardised score, z**, from a raw score, x, use the conversion $z = \frac{x-\mu}{\sigma}$ where μ is the mean and σ is the standard deviation of the raw scores. The standardised score is often called the **z-score.**

Example 6.1

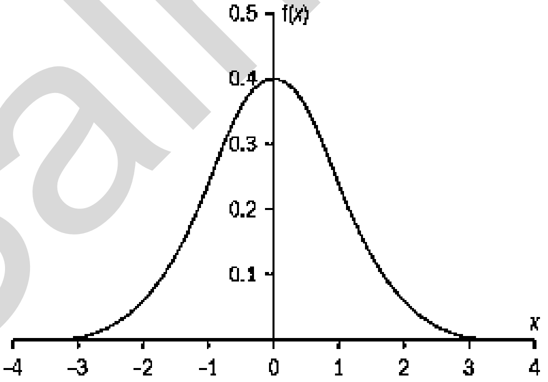
In her exams, Alexandra scores 75 in History and 87 in Maths. For the year group as a whole, History has a mean score of 63 in the examination with a standard deviation of 8, while Maths has a mean of 69 with a standard deviation of 15. Compare Alexandra's performance in these two subjects.

STANDARD NORMAL DISTRIBUTION

- The normal distribution is written as $X \sim N(\mu, \sigma^2)$.

This means 'X is distributed as a normal **random variable** with mean μ and variance σ^2'.

Since all normal distributions are the same basic shape, we only need to have probabilities for one particular case to allow us to calculate probabilities for all cases.



The **standard normal distribution** has mean 0 and variance 1.

The variable Z is often used for the standard normal distribution.

- For the standard normal distribution, $Z \sim N(0, 1^2)$ or $Z \sim N(0, 1)$.

By converting values to the standard normal distribution, you can use probability tables to calculate probabilities for any normal distribution.

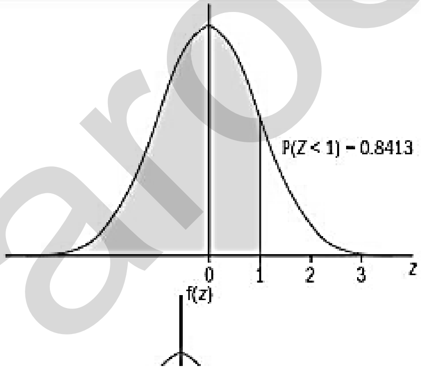
Be careful to distinguish between variance and standard deviation – if you are given N(83, 16), then you need to use the standard deviation of 4.

The tables give you $P(Z \leq z) = P(Z < z)$, where z is any value from 0 to 3 (beyond that the probabilities have become so small they are negligible).

Note: since the normal distribution is continuous the probability of particular values is zero, so it does not make any difference whether the inequality is strict or not.

To find $P(Z \leq 1)$:

Locate $z = 1.0$ in the table.
Go along to column 0, and read off the value below:
$P(Z \leq 1) = 0.8413$.



Because the normal distribution is symmetrical, the tables only give you half the set of probabilities – the other half are identical.

If you have a z-score which is not listed, such as $z = \frac{5}{3}$, you should calculate it to 3 decimal places and use that value.

$\frac{5}{3} = 1.6666\ldots = 1.667$ (3 d.p.)

$\Phi(1.667) = 0.9522$

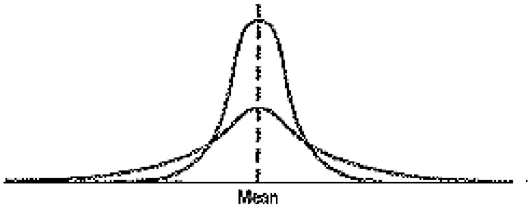
There is a second set of tables provided which give the exact z-scores for a limited number of **tail probabilities**.
These enable us to work out the z-score corresponding to a particular proportion.

For example, consider the z-value corresponding to the top 5%.

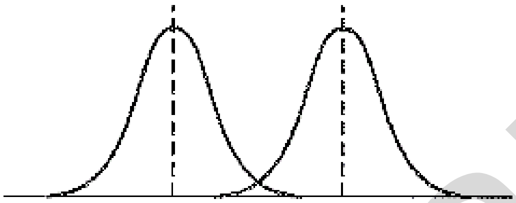
p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

There is a special symmetrical distribution known as the normal distribution. This is bell-shaped, centered around the mean.

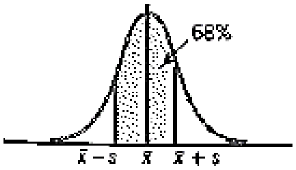
Here are two normal distribution with the same mean, but different standard deviations.



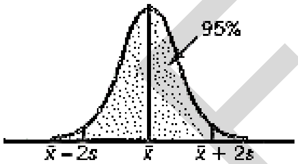
There are two normal distributions with the same standard deviation but with different means.



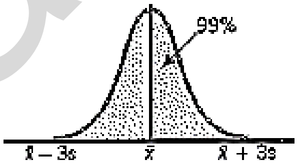
In a normal distribution:



Approximately 68% of the distribution lies within one standard deviation (s) of the mean.



Approximately 95% of the distribution lies within two standard deviations ($2s$) of the mean.



Over 99% of the distribution (nearly all!) lies within three standard deviations ($3s$) of the mean.

Example 6.2

All of these questions relate to the standard normal distribution, i.e. $Z \sim N(0, 1^2)$.

1. Find **a)** $\Phi(-0.06)$ **b)** $\Phi(2.63)$ **c)** $\Phi\left(\frac{4}{5}\right)$ **d)** $\Phi(2.5)-\Phi(1.2)$ **e)** $\Phi(1.43)-\Phi(-1.03)$.

If $Z \sim N(0, 1^2)$ find

a) $P(Z<1.62)$ **b)** $P(Z>0.76)$ **c)** $P(Z<-1.32)$ **d)** $P(-1.2<Z<1.7)$.

Example 6.3

(a) $P(0.345 < Z < 1.751)$
(b) $P(-1.41 < Z < -0.6)$
(c) $P(-1.4 < Z < -0.6)$
(d) $P(|Z| < 1.433)$

Exercise 7a Finding probabilities, where $Z \sim N(0, 1)$

Draw sketches to illustrate your answer and consider whether your answer is sensible.

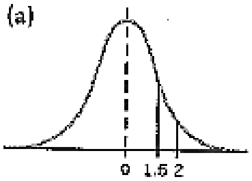
1. If $Z \sim N(0, 1)$, find
   (a) $P(Z < 0.874)$, (b) $P(Z > -0.874)$,
   (c) $P(Z > 0.874)$, (d) $P(Z < -0.874)$.

2. If $Z \sim N(0, 1)$, find
   (a) $P(Z > 1.8)$, (b) $P(Z < -0.65)$,
   (c) $P(Z > -2.46)$, (d) $P(Z < 1.36)$,
   (e) $P(Z > 2.58)$, (f) $P(Z > -2.37)$,
   (g) $P(Z < 1.86)$, (h) $P(Z < -0.725)$,
   (i) $P(Z > 1.863)$, (j) $P(Z < 1.63)$,
   (k) $P(Z > -2.061)$, (l) $P(Z < -2.875)$.

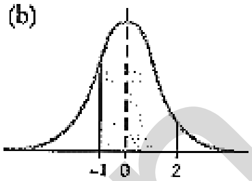
3. If $Z \sim N(0, 1)$, find
   (a) $P(Z > 1.645)$, (b) $P(Z < -1.645)$,
   (c) $P(Z > 1.282)$, (d) $P(Z > 1.96)$,
   (e) $P(Z > 2.575)$, (f) $P(Z > 2.326)$,
   (g) $P(Z > 2.808)$, (h) $P(Z < 1.96)$.

4. $Z \sim N(0, 1)$
   Find the probabilities represented by the shaded areas in the diagrams.

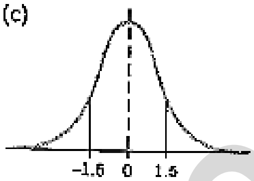
(a)



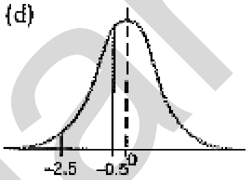
(b)



(c)

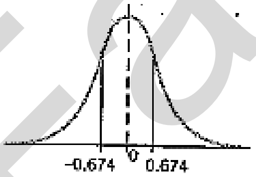


(d)



5. If $Z \sim N(0, 1)$, find
   (a) $P(0.829 < Z < 1.834)$,
   (b) $P(-2.56 < Z < 0.134)$,
   (c) $P(-1.762 < Z < -0.246)$,
   (d) $P(0 < Z < 1.73)$,
   (e) $P(-2.05 < Z < 0)$,
   (f) $P(-2.08 < Z < 2.08)$,
   (g) $P(1.764 < Z < 2.567)$,
   (h) $P(-1.65 < Z < 1.725)$,
   (i) $P(-0.98 < Z < -0.16)$,
   (j) $P(Z < -1.97$ or $Z > 2.5)$,
   (k) $P(|Z| < 1.78)$,
   (l) $P(|Z| > 0.754)$,
   (m) $P(-1.645 < Z < 1.645)$,
   (n) $P(|Z| > 2.326)$.

6. $Z \sim N(0, 1)$



Complete this statement:
The central ...% of the distribution lies between ±0.674.

7. $Z \sim N(0, 1)$ and $P(Z < a) = 0.3$, $P(a < Z < b) = 0.6$.
   Find
   (a) $P(Z < b)$,
   (b) $P(Z > a)$.

8. $Z \sim N(0, 1)$ and $P(Z < a) = 0.7$, $P(Z > b) = 0.45$.
   Find
   (a) $\Phi(b)$,
   (b) $P(b < Z < a)$.

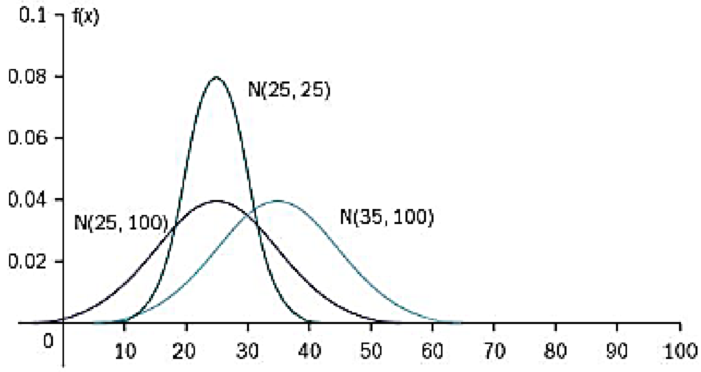
9. $Z \sim N(0, 1)$ and $P(|Z| < a) = 0.8$.
   Find
   (a) $P(Z < a)$,
   (b) $P(Z > a)$.

Exercise 7a Finding probabilities, where $Z \sim N(0, 1)$ (page 367)

1. (a) 0.8089 (b) 0.8089 (c) 0.1911 (d) 0.1911
2. (a) 0.0359 (b) 0.2578 (c) 0.9931 (d) 0.9131
   (e) 0.0049 (f) 0.9911 (g) 0.9686 (h) 0.2343
   (i) 0.0312 (j) 0.9484 (k) 0.9803 (l) 0.0021
3. (a) 0.05 (b) 0.05 (c) 0.0999 (d) 0.025 (e) 0.005
   (f) 0.01 (g) 0.0025 (h) 0.075
4 (a) 0.044 (b) 0.8185 (c) 0.1336 (d) 0.3023
5. (a) 0.1703 (b) 0.5481 (c) 0.3639 (d) 0.4582
   (e) 0.4798 (f) 0.9624 (g) 0.0337 (h) 0.9082
   (i) 0.2729 (j) 0.030 (k) 0.925 (l) 0.4508
   (m) 0.9 (n) 0.02
6. 50%
7. (a) 0.9 (b) 0.7
8. (a) 0.55 (b) 0.15
9. (a) 0.9 (b) 0.1

CALCULATING PROBABILITIES FOR THE N(μ, σ^2) DISTRBUTION

All normal distributions are essentially the same shape – they may have a different centre, or be more peaked, but they can all be standardised to the N(0, 1) distribution.



For a distribution $X \sim N(\mu, \sigma^2)$, we can find the probability of X taking a range of values through the following.

- First calculate z by using $z = \frac{x-\mu}{\sigma}$.
- Then use the table of probabilities to find $\Phi(z)$.
- **Determine** the probability required, referring to a sketch.

Example 6.4

Lengths of metal strips produced by a machine are normally distributed with mean length of 150 cm and a standard deviation of 10 cm.
Find the probability that the length of a randomly selected strip is
(a) shorter than 165 cm,
(b) within 5 cm of the mean.

Example 6.5

The time taken by the milkman to deliver to the High Street is normally distributed with a mean of 12 minutes and a standard deviation of 2 minutes. He delivers milk every day. Estimate the number of days during the year when he takes
(a) longer than 17 minutes,
(b) less than ten minutes.
(c) between nine and 13 minutes.

USING THE STANDARD NORMAL TABLES IN REVERSE TO FIND z WHEN $\Phi(z)$ IS KNOWN

Example 6.6

If Z ~ N (0,1), find the value of a if

(a) P (Z < a) = 0.9693

(b) P (Z > a) = 0.3802

(c) P (Z > a) = 0.7367

(d) P (Z < a) = 0.0793

Example 6.7

If Z ~ N (0,1) find a such that P (|Z| < a) = 0.9

USING THE TABLES IN REVERSE FOR ANY NORMAL VARIABLE X

Example 6.8

The heights of female students at a particular college are normally distributed with a mean of 169cm and a standard deviation of 9cm.

(a) Given that 80% of these female students have a height less than h cm, find the value of h.

(b) Given that 60% of these females students have a height greater than s cm, find the value of s.

Example 6.9

The marks of 500 candidates in an examination are normally distributed with a mean of 45 marks and a standard deviation of 20 marks.

(a) Given that the pass mark is 41, estimates the number of candidates who passed the examination.

(b) If 5% of the candidates obtain a distinction by scoring x marks or more, estimate the value of x.

(c) Estimate the interquartile range of the distribution.

FINDING THE VALUE OF μ OR σ OR BOTH

Example 6.10

The lengths of certain items follow a normal distribution with mean μ cm and standard deviation 6 cm. It is known that 4.78% of the items have length greater than 82 cm. Find the value of the mean μ.

Example 6.11

$X \sim N(100, \sigma^2)$ and $P(X < 106) = 0.8849$.
Find the value of the standard deviation σ.

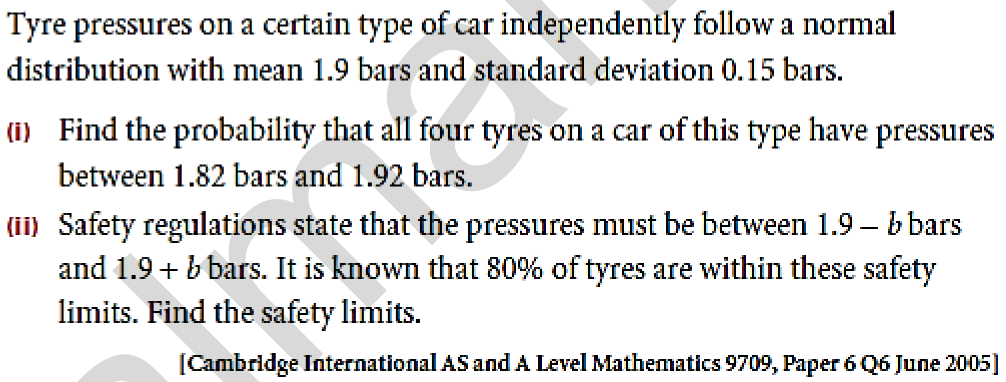
Example 6.12

The masses of boxes of oranges are normally distributed such that 30% of them are greater than 4.00 kg and 20% are greater than 4.53 kg. Estimate the mean and standard deviation of the masses.

Example 6.13

The speeds of cars passing a certain point on a motorway can be taken to be normally distributed. Observations show that of cars passing the point, 95 % are travelling at less than 85 m.p.h. and 10% are travelling at less than 55 m.p.h.
(a) Find the average speed of the cars passing the point.
(b) Find the proportion of cars that travel at more than 70 m.p.h.

Example 6.14



Tyre pressures on a certain type of car independently follow a normal distribution with mean 1.9 bars and standard deviation 0.15 bars.

(i) Find the probability that all four tyres on a car of this type have pressures between 1.82 bars and 1.92 bars.

(ii) Safety regulations state that the pressures must be between $1.9 - b$ bars and $1.9 + b$ bars. It is known that 80% of tyres are within these safety limits. Find the safety limits.

[Cambridge International AS and A Level Mathematics 9709, Paper 6 Q6 June 2005]

Example 6.15

The lengths of fish of a certain type have a normal distribution with mean 38 cm. It is found that 5% of the fish are longer than 50 cm.

(i) Find the standard deviation.

(ii) When fish are chosen for sale, those shorter than 30 cm are rejected. Find the proportion of fish rejected.

(iii) 9 fish are chosen at random. Find the probability that at least one of them is longer than 50 cm.

[Cambridge International AS and A Level Mathematics 9709, Paper 6 Q3 June 2006]

Example 6.16

(i) The random variable X is normally distributed. The mean is twice the standard deviation. It is given that $P(X > 5.2) = 0.9$. Find the standard deviation.

(ii) A normal distribution has mean μ and standard deviation σ. If 800 observations are taken from this distribution, how many would you expect to be between $\mu - \sigma$ and $\mu + \sigma$?

[Cambridge International AS and A Level Mathematics 9709, Paper 6 Q3 June 2007]

Example 6.17

In a certain country the time taken for a common infection to clear up is normally distributed with mean μ days and standard deviation 2.6 days. 25% of these infections clear up in less than 7 days.

(i) Find the value of μ.

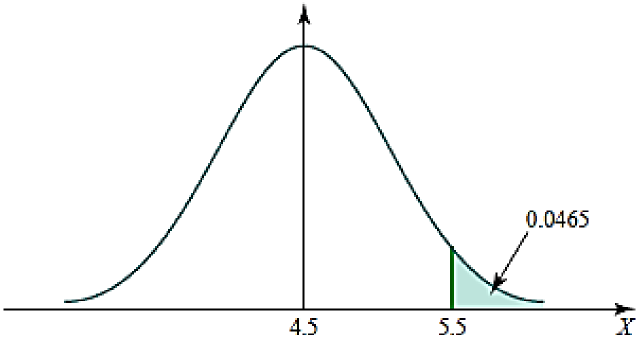
In another country the standard deviation of the time taken for the infection to clear up is the same as in part **(i)** but the mean is 6.5 days. The time taken is normally distributed.

(ii) Find the probability that, in a randomly chosen case from this country, the infection takes longer than 6.2 days to clear up.

[Cambridge International AS and A Level Mathematics 9709, Paper 6 Q4 June 2008]

Example 6.18

The random variable X has a normal distribution with mean 4.5. It is given that $P(X > 5.5) = 0.0465$ (see diagram).



(i) Find the standard deviation of X.

(ii) Find the probability that a random observation of X lies between 3.8 and 4.8.

[Cambridge International AS and A Level Mathematics 9709, Paper 6 Q4 November 2007]

Example 6.19

(i) The daily minimum temperature in degrees Celsius (°C) in January in Ottawa is a random variable with distribution $N(-15.1, 62.0)$. Find the probability that a randomly chosen day in January in Ottawa has a minimum temperature above 0°C.

(ii) In another city the daily minimum temperature in °C in January is a random variable with distribution $N(\mu, 40.0)$. In this city the probability that a randomly chosen day in January has a minimum temperature above 0°C is 0.8888. Find the value of μ.

[Cambridge International AS and A Level Mathematics 9709, Paper 6 Q3 November 2008]

Example 6.20

The times for a certain car journey have a normal distribution with mean 100 minutes and standard deviation 7 minutes. Journey times are classified as follows:

'short'	(the shortest 33% of times),
'long'	(the longest 33% of times),
'standard'	(the remaining 34% of times).

(i) Find the probability that a randomly chosen car journey takes between 85 and 100 minutes.

(ii) Find the least and greatest times for 'standard' journeys.

[Cambridge International AS and A Level Mathematics 9709, Paper 61 Q3 November 2009]

SELECTED PAST PAPER QUESTIONS

Question 1

N13/62/Q1

It is given that $X \sim \mathrm{N}(1.5, 3.2^2)$. Find the probability that a randomly chosen value of X is less than -2.4. [3]

Question 2

N12/61/Q3

Lengths of rolls of parcel tape have a normal distribution with mean 75 m, and 15% of the rolls have lengths less than 73 m.

(i) Find the standard deviation of the lengths. [3]

Alison buys 8 rolls of parcel tape.

(ii) Find the probability that fewer than 3 of these rolls have lengths more than 77 m. [3]

HOMEWORK: NORMAL DISTRIBUTION – VARIANT 62

1 **(i)** The height of sunflowers follows a normal distribution with mean 112 cm and standard deviation 17.2 cm. Find the probability that the height of a randomly chosen sunflower is greater than 120 cm. [3]

(ii) When a new fertiliser is used, the height of sunflowers follows a normal distribution with mean 115 cm. Given that 80% of the heights are now greater than 103 cm, find the standard deviation. [3]

Answers: (i) 0.321; (ii) 14.3 . **J03/Q3**

2 Melons are sold in three sizes: small, medium and large. The weights follow a normal distribution with mean 450 grams and standard deviation 120 grams. Melons weighing less than 350 grams are classified as small.

(i) Find the proportion of melons which are classified as small. [3]

(ii) The rest of the melons are divided in equal proportions between medium and large. Find the weight above which melons are classified as large. [5]

Answers: (i) 0.203; (ii) 481. **J04/Q4**

3 Tyre pressures on a certain type of car independently follow a normal distribution with mean 1.9 bars and standard deviation 0.15 bars.

(i) Find the probability that all four tyres on a car of this type have pressures between 1.82 bars and 1.92 bars. [5]

(ii) Safety regulations state that the pressures must be between $1.9 - b$ bars and $1.9 + b$ bars. It is known that 80% of tyres are within these safety limits. Find the safety limits. [3]

Answers: (i) 0.00429; (ii) 1.71 to 2.09. **J05/Q6**

4 The lengths of fish of a certain type have a normal distribution with mean 38 cm. It is found that 5% of the fish are longer than 50 cm.

(i) Find the standard deviation. [3]

(ii) When fish are chosen for sale, those shorter than 30 cm are rejected. Find the proportion of fish rejected. [3]

(iii) 9 fish are chosen at random. Find the probability that at least one of them is longer than 50 cm. [2]

Answers: (i) 7.29; (ii) 0.136; (iii) 0.370. **J06/Q3**

5 **(a)** The random variable X is normally distributed. The mean is twice the standard deviation. It is given that $P(X > 5.2) = 0.9$. Find the standard deviation. [4]

(b) A normal distribution has mean μ and standard deviation σ. If 800 observations are taken from this distribution, how many would you expect to be between $\mu - \sigma$ and $\mu + \sigma$? [3]

Answers: (a) 7.24; (b) 546. **J07/Q3**

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6 In a certain country the time taken for a common infection to clear up is normally distributed with mean μ days and standard deviation 2.6 days. 25% of these infections clear up in less than 7 days.

(i) Find the value of μ. [4]

In another country the standard deviation of the time taken for the infection to clear up is the same as in part **(i)**, but the mean is 6.5 days. The time taken is normally distributed.

(ii) Find the probability that, in a randomly chosen case from this country, the infection takes longer than 6.2 days to clear up. [3]

Answers: (i) 8.75; (ii) 0.546. **J08/Q4**

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7 The volume of milk in millilitres in cartons is normally distributed with mean μ and standard deviation 8. Measurements were taken of the volume in 900 of these cartons and it was found that 225 of them contained more than 1002 millilitres.

(i) Calculate the value of μ. [3]

(ii) Three of these 900 cartons are chosen at random. Calculate the probability that exactly 2 of them contain more than 1002 millilitres. [2]

Answers: (i) 997; (ii) 0.140. **J09/Q1**

---

8 The lengths of new pencils are normally distributed with mean 11 cm and standard deviation 0.095 cm.

(i) Find the probability that a pencil chosen at random has a length greater than 10.9 cm. [2]

(ii) Find the probability that, in a random sample of 6 pencils, at least two have lengths less than 10.9 cm. [3]

Answers: (i) 0.854; (ii) 0.215. **J10/62/Q2**

---

9 The random variable X is normally distributed with mean μ and standard deviation σ.

(i) Given that $5\sigma = 3\mu$, find $P(X < 2\mu)$. [3]

(ii) With a different relationship between μ and σ, it is given that $P(X < \frac{1}{3}\mu) = 0.8524$. Express μ in terms of σ. [3]

Answers: (i) 0.952, (ii) -1.57σ.

J10/62/Q4

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10 The distance in metres that a ball can be thrown by pupils at a particular school follows a normal distribution with mean 35.0 m and standard deviation 11.6 m.

(i) Find the probability that a randomly chosen pupil can throw a ball between 30 and 40 m. [3]

(ii) The school gives a certificate to the 10% of pupils who throw further than a certain distance. Find the least distance that must be thrown to qualify for a certificate. [3]

\- *Answers*: (i) 0.334; (ii) 49.9.

N02/Q3

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11 In a normal distribution, 69% of the distribution is less than 28 and 90% is less than 35. Find the mean and standard deviation of the distribution. [6]

Answers: 8.91, 23.6.

N03/Q3

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12 The length of Paulo's lunch break follows a normal distribution with mean μ minutes and standard deviation 5 minutes. On one day in four, on average, his lunch break lasts for more than 52 minutes.

(i) Find the value of μ. [3]

(ii) Find the probability that Paulo's lunch break lasts for between 40 and 46 minutes on every one of the next four days. [4]

Answers: (i) 48.6; (ii) 0.00438.

N04/Q5

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13 In tests on a new type of light bulb it was found that the time they lasted followed a normal distribution with standard deviation 40.6 hours. 10% lasted longer than 5130 hours.

(i) Find the mean lifetime, giving your answer to the nearest hour. [3]

(ii) Find the probability that a light bulb fails to last for 5000 hours. [3]

(iii) A hospital buys 600 of these light bulbs. Using a suitable approximation, find the probability that fewer than 65 light bulbs will last longer than 5130 hours. [4]

Answers: (i) 5080; (ii) 0.0273; (iii) 0.730.

N05/Q7

---

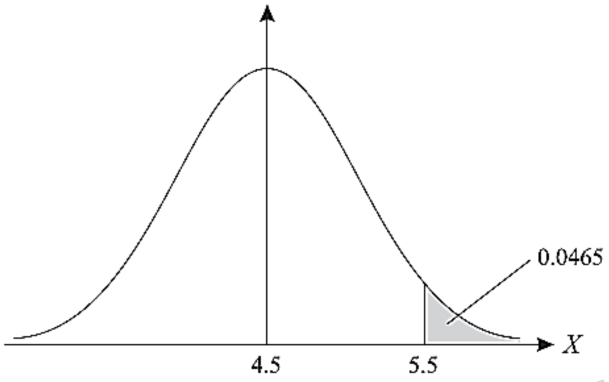
14 (i) Give an example of a variable in real life which could be modelled by a normal distribution. [1]

(ii) The random variable X is normally distributed with mean μ and variance 21.0. Given that $P(X > 10.0) = 0.7389$, find the value of μ. [3]

(iii) If 300 observations are taken at random from the distribution in part (ii), estimate how many of these would be greater than 22.0. [4]

Answers: (ii) 12.9; (iii) 7. **N06/Q5**

15



The random variable X has a normal distribution with mean 4.5. It is given that $P(X > 5.5) = 0.0465$ (see diagram).

(i) Find the standard deviation of X. [3]

(ii) Find the probability that a random observation of X lies between 3.8 and 4.8. [4]

Answers: (i) 0.595; (ii) 0.573. **N07/Q4**

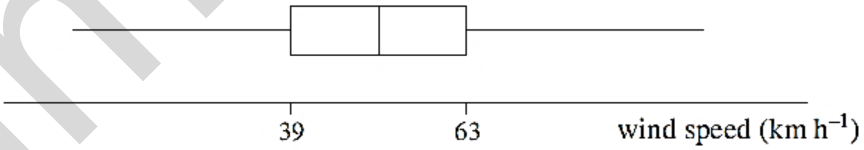
---

16 **(i)** The daily minimum temperature in degrees Celsius (°C) in January in Ottawa is a random variable with distribution $N(-15.1, 62.0)$. Find the probability that a randomly chosen day in January in Ottawa has a minimum temperature above 0 °C. [3]

(ii) In another city the daily minimum temperature in °C in January is a random variable with distribution $N(\mu, 40.0)$. In this city the probability that a randomly chosen day in January has a minimum temperature above 0 °C is 0.8888. Find the value of μ. [3]

Answers: (i) 0.0276; (ii) 7.72. **N08/Q3**

17



Measurements of wind speed on a certain island were taken over a period of one year. A box-and-whisker plot of the data obtained is displayed above, and the values of the quartiles are as shown. It is suggested that wind speed can be modelled approximately by a normal distribution with mean μ km h^{-1} and standard deviation σ km h^{-1}.

(i) Estimate the value of μ. [1]

(ii) Estimate the value of σ. [3]

Answers: (i) 51; (ii) 17.8. **N09/62/Q1**

---

19 The weights, X grams, of bars of soap are normally distributed with mean 125 grams and standard deviation 4.2 grams.

(i) Find the probability that a randomly chosen bar of soap weighs more than 128 grams. [3]

(ii) Find the value of k such that $\mathrm{P}(k < X < 128) = 0.7465$. [4]

(iii) Five bars of soap are chosen at random. Find the probability that more than two of the bars each weigh more than 128 grams. [4]

Answers: (i) 0.238; (ii) 116; (iii) 0.0909 or 0.0910. **N09/62/Q7**

---

20 The distance the Zotoc car can travel on 20 litres of fuel is normally distributed with mean 320 km and standard deviation 21.6 km. The distance the Ganmor car can travel on 20 litres of fuel is normally distributed with mean 350 km and standard deviation 7.5 km. Both cars are filled with 20 litres of fuel and are driven towards a place 367 km away.

(i) For each car, find the probability that it runs out of fuel before it has travelled 367 km. [3]

(ii) The probability that a Zotoc car can travel at least $(320 + d)$ km on 20 litres of fuel is 0.409. Find the value of d. [4]

Answers: (i) 0.985, 0.988; (ii) 4.97. **N10/62/Q5**

---

21 The lengths, in centimetres, of drinking straws produced in a factory have a normal distribution with mean μ and variance 0.64. It is given that 10% of the straws are shorter than 20 cm.

(i) Find the value of μ. [3]

(ii) Find the probability that, of 4 straws chosen at random, fewer than 2 will have a length between 21.5 cm and 22.5 cm. [6]

Answers: (i) 21.0; (ii) 0.746. **J11/62/Q6**

22 The times taken to play Beethoven's Sixth Symphony can be assumed to have a normal distribution with mean 41.1 minutes and standard deviation 3.4 minutes. Three occasions on which this symphony is played are chosen at random.

(i) Find the probability that the symphony takes longer than 42 minutes to play on exactly 1 of these occasions. [4]

The times taken to play Beethoven's Fifth Symphony can also be assumed to have a normal distribution. The probability that the time is less than 26.5 minutes is 0.1, and the probability that the time is more than 34.6 minutes is 0.05.

(ii) Find the mean and standard deviation of the times to play this symphony. [5]

(iii) Assuming that the times to play the two symphonies are independent of each other, find the probability that, when both symphonies are played, both of the times are less than 34.6 minutes. [4]

Answers: (i) 0.433 (ii) μ = 30.0, σ= 2.77 (iii) 0.0266 **J12/62/Q7**

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23 The random variable Y is normally distributed with mean equal to five times the standard deviation. It is given that $P(Y > 20) = 0.0732$. Find the mean. [3]

Answer: 15.5 **J13/62/Q1**

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24 Cans of lemon juice are supposed to contain 440 ml of juice. It is found that the actual volume of juice in a can is normally distributed with mean 445 ml and standard deviation 3.6 ml.

(i) Find the probability that a randomly chosen can contains less than 440 ml of juice. [3]

It is found that 94% of the cans contain between $(445 - c)$ ml and $(445 + c)$ ml of juice.

(ii) Find the value of c. [3]

Answer: 0.0824 *Answer*: 6.77 **J13/62/Q3**

---

25 The time Rafa spends on his homework each day in term-time has a normal distribution with mean 1.9 hours and standard deviation σ hours. On 80% of these days he spends more than 1.35 hours on his homework.

(i) Find the value of σ. [3]

(ii) Find the probability that, on a randomly chosen day in term-time, Rafa spends less than 2 hours on his homework. [2]

Answers: 0.653, 0.561, **J14/62/Q7**

26 **(a)** Once a week Zak goes for a run. The time he takes, in minutes, has a normal distribution with mean 35.2 and standard deviation 4.7.

(i) Find the expected number of days during a year (52 weeks) for which Zak takes less than 30 minutes for his run. [4]

(ii) The probability that Zak's time is between 35.2 minutes and t minutes, where $t > 35.2$, is 0.148. Find the value of t. [3]

(b) The random variable X has the distribution $N(\mu, \sigma^2)$. It is given that $P(X < 7) = 0.2119$ and $P(X < 10) = 0.6700$. Find the values of μ and σ. [5]

Answers: (a)(i) 6.99, (ii) 37.0, (b) $\mu = 8.94$, $\sigma = 2.42$ **J15/62/Q7**

---

27 **(i)** In a certain country, the daily minimum temperature, in °C, in winter has the distribution N(8, 24). Find the probability that a randomly chosen winter day in this country has a minimum temperature between 7 °C and 12 °C. [3]

The daily minimum temperature, in °C, in another country in winter has a normal distribution with mean μ and standard deviation 2μ.

(ii) Find the proportion of winter days on which the minimum temperature is below zero. [2]

(iii) 70 winter days are chosen at random. Find how many of these would be expected to have a minimum temperature which is more than three times the mean. [3]

(iv) The probability of the minimum temperature being above 6 °C on any winter day is 0.0735. Find the value of μ. [3]

Answers: (i) 0.373; (ii) 0.309; (iii) 11.1; (iv) 1.54 **N11/62/Q7**

---

28 The random variable X is the daily profit, in thousands of dollars, made by a company. X is normally distributed with mean 6.4 and standard deviation 5.2.

(i) Find the probability that, on a randomly chosen day, the company makes a profit between \$10 000 and \$12 000. [3]

(ii) Find the probability that the company makes a loss on exactly 1 of the next 4 consecutive days. [4]

Answers: (i) 0.104; (ii) 0.309. **N12/62/Q2**

---

29 It is given that $X \sim N(1.5, 3.2^2)$. Find the probability that a randomly chosen value of X is less than –2.4. [3]

Answer: 0.111 **N13/62/Q1**

30 The amount of fibre in a packet of a certain brand of cereal is normally distributed with mean 160 grams. 19% of packets of cereal contain more than 190 grams of fibre.

(i) Find the standard deviation of the amount of fibre in a packet. [3]

(ii) Kate buys 12 packets of cereal. Find the probability that at least 1 of the packets contains more than 190 grams of fibre. [2]

Answers: 34.2, 0.920 **N13/62/Q3**

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31 **(a)** The time, X hours, for which people sleep in one night has a normal distribution with mean 7.15 hours and standard deviation 0.88 hours.

(i) Find the probability that a randomly chosen person sleeps for less than 8 hours in a night. [2]

(ii) Find the value of q such that $P(X < q) = 0.75$. [3]

(b) The random variable Y has the distribution $N(\mu, \sigma^2)$, where $2\sigma = 3\mu$ and $\mu \neq 0$. Find $P(Y > 4\mu)$. [3]

Answers: 0.833, 7.74, 0.0228 **N14/62/Q5**

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32 **(a)** A petrol station finds that its daily sales, in litres, are normally distributed with mean 4520 and standard deviation 560.

(i) Find on how many days of the year (365 days) the daily sales can be expected to exceed 3900 litres. [4]

The daily sales at another petrol station are X litres, where X is normally distributed with mean m and standard deviation 560. It is given that $P(X > 8000) = 0.122$.

(ii) Find the value of m. [3]

(iii) Find the probability that daily sales at this petrol station exceed 8000 litres on fewer than 2 of 6 randomly chosen days. [3]

(b) The random variable Y is normally distributed with mean μ and standard deviation σ. Given that $\sigma = \frac{2}{3}\mu$, find the probability that a random value of Y is less than 2μ. [3]

Answers: (i) 315 or 316, (ii) 7350, (iii) 0.840, (iv) 0.933 **N15/62/Q7**

HOMEWORK: NORMAL DISTRIBUTION VARIANT 61 AND 63

1 The lengths, in metres, of cars in a city are normally distributed with mean μ and standard deviation 0.714. The probability that a randomly chosen car has a length more than 3.2 metres and less than μ metres is 0.475. Find μ. [4]

Answer: 4.60 **61/J15/1**

2 The weights, in grams, of onions in a supermarket have a normal distribution with mean μ and standard deviation 22. The probability that a randomly chosen onion weighs more than 195 grams is 0.128. Find the value of μ. [3]

Answer: 170 **63/J15/2**

3 The heights of books in a library, in cm, have a normal distribution with mean 21.7 and standard deviation 6.5. A book with a height of more than 29 cm is classified as 'large'.

(i) Find the probability that, of 8 books chosen at random, fewer than 2 books are classified as large. [6]

(ii) n books are chosen at random. The probability of there being at least 1 large book is more than 0.98. Find the least possible value of n. [3]

Answer : i) 0.718 ii) 28 **63/J15/5**

4 The random variable X has the distribution $N(\mu, \sigma^2)$. It is given that $P(X < 54.1) = 0.5$ and $P(X > 50.9) = 0.8665$. Find the values of μ and σ. [4]

Answer: 54.1 , 2.88 **61/N15/2**

5 Amy's friend Marok measured her pulse rate every day after running for half an hour. Marok's pulse rate, in beats per minute, was found to have a mean of 148.6 and a standard deviation of 18.5. Assuming that pulse rates have a normal distribution, find what proportion of Marok's pulse rates, after running for half an hour, were above 160 beats per minute. [3]

Answer: 0.269 **61/N15/4**

6 The time taken for cucumber seeds to germinate under certain conditions has a normal distribution with mean 125 hours and standard deviation σ hours.

(i) It is found that 13% of seeds take longer than 136 hours to germinate. Find the value of σ. [3]

(ii) 170 seeds are sown. Find the expected number of seeds which take between 131 and 141 hours to germinate. [4]

Answer: i) 9.76 ii) 37 **63/N15/4**

7 Packets of tea are labelled as containing 250 g. The actual weight of tea in a packet has a normal distribution with mean 260 g and standard deviation σ g. Any packet with a weight less than 250 g is classed as 'underweight'. Given that 1% of packets of tea are underweight, find the value of σ. [3]

Answer: 4.30 **63/N14/1**

8 Gem stones from a certain mine have weights, X grams, which are normally distributed with mean 1.9 g and standard deviation 0.55 g. These gem stones are sorted into three categories for sale depending on their weights, as follows.

Small: under 1.2 g Medium: between 1.2 g and 2.5 g Large: over 2.5 g

(i) Find the proportion of gem stones in each of these three categories. [5]

(ii) Find the value of k such that $\mathrm{P}(k < X < 2.5) = 0.8$. [4]

Answer: i) 0.1014, 0.138, 0.761 ii) 1.06 **63/N14/5**

9 **(a)** The time, X hours, for which people sleep in one night has a normal distribution with mean 7.15 hours and standard deviation 0.88 hours.

(i) Find the probability that a randomly chosen person sleeps for less than 8 hours in a night. [2]

(ii) Find the value of q such that $\mathrm{P}(X < q) = 0.75$. [3]

(b) The random variable Y has the distribution $\mathrm{N}(\mu, \sigma^2)$, where $2\sigma = 3\mu$ and $\mu \neq 0$. Find $\mathrm{P}(Y > 4\mu)$. [3]

Answer: a) 0.833 ii) 7.74 b) 0.0228 **62/N14/5**

10 A farmer finds that the weights of sheep on his farm have a normal distribution with mean 66.4 kg and standard deviation 5.6 kg.

(i) 250 sheep are chosen at random. Estimate the number of sheep which have a weight of between 70 kg and 72.5 kg. [5]

(ii) The proportion of sheep weighing less than 59.2 kg is equal to the proportion weighing more than y kg. Find the value of y. [2]

Another farmer finds that the weights of sheep on his farm have a normal distribution with mean μ kg and standard deviation 4.92 kg. 25% of these sheep weigh more than 67.5 kg.

(iii) Find the value of μ. [3]

- **Answer: i) 30 or 31, ii) 73.6 iii) 64.2** **61/N14/6**

11 When Moses makes a phone call, the amount of time that the call takes has a normal distribution with mean 6.5 minutes and standard deviation 1.76 minutes.

(i) 90% of Moses's phone calls take longer than t minutes. Find the value of t. [3]

(ii) Find the probability that, in a random sample of 9 phone calls made by Moses, more than 7 take a time which is within 1 standard deviation of the mean. [5]

Answer: i) 4.24 , 0.167 **63/J14/5**

12 The petrol consumption of a certain type of car has a normal distribution with mean 24 kilometres per litre and standard deviation 4.7 kilometres per litre. Find the probability that the petrol consumption of a randomly chosen car of this type is between 21.6 kilometres per litre and 28.7 kilometres per litre. [4]

Answer: 0.537 **61/J14/1**

13 Lengths of a certain type of white radish are normally distributed with mean μ cm and standard deviation σ cm. 4% of these radishes are longer than 12 cm and 32% are longer than 9 cm. Find μ and σ. [5]

Answer: 7.91, 2.34 **61/J14/2**

14 A factory produces flower pots. The base diameters have a normal distribution with mean 14 cm and standard deviation 0.52 cm. Find the probability that the base diameters of exactly 8 out of 10 randomly chosen flower pots are between 13.6 cm and 14.8 cm. [5]

Answer: i) 0.252 **63/N13/2**

15 **(a)** The random variable X is normally distributed with mean 82 and standard deviation 7.4. Find the value of q such that $P(82 - q < X < 82 + q) = 0.44$. [3]

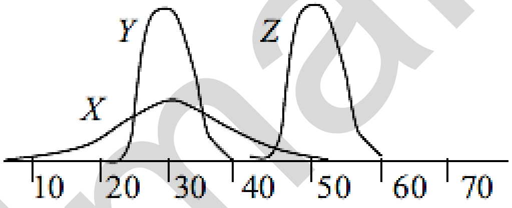
(b) The random variable Y is normally distributed with mean μ and standard deviation σ. It is given that $5\mu = 2\sigma^2$ and that $P(Y < \frac{1}{2}\mu) = 0.281$. Find the values of μ and σ. [4]

Answer: a) 4.31 b) 2.90 or 3.36 **63/N13/5**

16 It is given that $X \sim N(30, 49)$, $Y \sim N(30, 16)$ and $Z \sim N(50, 16)$. On a single diagram, with the horizontal axis going from 0 to 70, sketch three curves to represent the distributions of X, Y and Z. [3]

61/N13/1

Answer:



17 Buildings in a certain city centre are classified by height as tall, medium or short. The heights can be modelled by a normal distribution with mean 50 metres and standard deviation 16 metres. Buildings with a height of more than 70 metres are classified as tall.

(i) Find the probability that a building chosen at random is classified as tall. [2]

(ii) The rest of the buildings are classified as medium and short in such a way that there are twice as many medium buildings as there are short ones. Find the height below which buildings are classified as short. [5]

18 **Answer: i) 0.106 ii)41.5** **63/J13/3**

19 **(a)** The random variable Y is normally distributed with positive mean μ and standard deviation $\frac{1}{2}\mu$. Find the probability that a randomly chosen value of Y is negative. [3]

(b) The weights of bags of rice are normally distributed with mean 2.04 kg and standard deviation σ kg. In a random sample of 8000 such bags, 253 weighed over 2.1 kg. Find the value of σ. [4]

Answer: i) 0.0228 ii) 0.0323 **61/J13/4**

20 In a normal distribution with mean 9.3, the probability of a randomly chosen value being greater than 5.6 is 0.85. Find the standard deviation. [3]

Answer: i) 3.57 **63/N12/1**

21 The random variable X is such that $X \sim \mathrm{N}(82, 126)$.

(i) A value of X is chosen at random and rounded to the nearest whole number. Find the probability that this whole number is 84. [3]

(ii) Five independent observations of X are taken. Find the probability that at most one of them is greater than 87. [4]

(iii) Find the value of k such that $\mathrm{P}(87 < X < k) = 0.3$. [5]

Answer: i) 0.0350 ii) 0.471 iii) 103 **63/N12/5**

22 * Lengths of rolls of parcel tape have a normal distribution with mean 75 m, and 15% of the rolls have lengths less than 73 m.

(i) Find the standard deviation of the lengths. [3]

Alison buys 8 rolls of parcel tape.

(ii) Find the probability that fewer than 3 of these rolls have lengths more than 77 m. [3]

Answer: i) 1.93 ii) 0.895 **61/N12/3**

23 The lengths, in cm, of trout in a fish farm are normally distributed. 96% of the lengths are less than 34.1 cm and 70% of the lengths are more than 26.7 cm.

(i) Find the mean and the standard deviation of the lengths of the trout. [5]

In another fish farm, the lengths of salmon, X cm, are normally distributed with mean 32.9 cm and standard deviation 2.4 cm.

(ii) Find the probability that a randomly chosen salmon is 34 cm long, correct to the nearest centimetre. [3]

(iii) Find the value of t such that $\mathrm{P}(31.8 < X < t) = 0.5$. [4]

Answer: i) 28.4, 3.25 ii) 0.149 iii) 35.1 **63/J12/6**

24 It is given that $X \sim N(28.3, 4.5)$. Find the probability that a randomly chosen value of X lies between 25 and 30. [3]

Answer: 0.729 **61/J12/1**

25 The lengths of body feathers of a particular species of bird are modelled by a normal distribution. A researcher measures the lengths of a random sample of 600 body feathers from birds of this species and finds that 63 are less than 6 cm long and 155 are more than 12 cm long.

(i) Find estimates of the mean and standard deviation of the lengths of body feathers of birds of this species. [5]

(ii) In a random sample of 1000 body feathers from birds of this species, how many would the researcher expect to find with lengths more than 1 standard deviation from the mean? [4]

Answer: i) 9.9, 3.15 ii) 317 **61/J12/6**

26 The random variable X is normally distributed and is such that the mean μ is three times the standard deviation σ. It is given that $P(X < 25) = 0.648$.

(i) Find the values of μ and σ. [4]

(ii) Find the probability that, from 6 random values of X, exactly 4 are greater than 25. [2]

Answer: i) 22.2, 7.40 ii) 0.0967 **63/N11/1**

27 The weights of letters posted by a certain business are normally distributed with mean 20 g. It is found that the weights of 94% of the letters are within 12 g of the mean.

(i) Find the standard deviation of the weights of the letters. [3]

(ii) Find the probability that a randomly chosen letter weighs more than 13 g. [3]

(iii) Find the probability that at least 2 of a random sample of 7 letters have weights which are more than 12 g above the mean. [3]

Answer: i) 6.38 ii) 0.864 iii) 0.0171 **61/N11/5**

28 The random variable X is normally distributed with mean μ and standard deviation $\frac{1}{4}\mu$. It is given that $P(X > 20) = 0.04$.

(i) Find μ. [3]

(ii) Find $P(10 < X < 20)$. [3]

(iii) 250 independent observations of X are taken. Find the probability that at least 235 of them are less than 20. [5]

Answer: i) 13.9 ii) 0.829 iii) 0.962 **63/J11/5**

29 **(a)** The random variable X is normally distributed with mean μ and standard deviation σ. It is given that $3\mu = 7\sigma^2$ and that $P(X > 2\mu) = 0.1016$. Find μ and σ. [4]

(b) It is given that $Y \sim N(33, 21)$. Find the value of a given that $P(33 - a < Y < 33 + a) = 0.5$. [4]

Answer: i) 0.693, 0.545 ii) 3.09

30 Name the distribution and suggest suitable numerical parameters that you could use to model the weights in kilograms of female 18-year-old students. [2]

Answer: Normal, Any sensible values **63/N10/1**

31 The times spent by people visiting a certain dentist are independent and normally distributed with a mean of 8.2 minutes. 79% of people who visit this dentist have visits lasting less than 10 minutes.

(i) Find the standard deviation of the times spent by people visiting this dentist. [3]

(ii) Find the probability that the time spent visiting this dentist by a randomly chosen person deviates from the mean by more than 1 minute. [3]

(iii) Find the probability that, of 6 randomly chosen people, more than 2 have visits lasting longer than 10 minutes. [3]

Answer: i) 2.23 ii) 0.654 iii) 0.112 **63/N10/7**

32 On average, 2 apples out of 15 are classified as being underweight. Find the probability that in a random sample of 200 apples, the number of apples which are underweight is more than 21 and less than 35. [5]

Answer: 0.807

33 The times taken by students to get up in the morning can be modelled by a normal distribution with mean 26.4 minutes and standard deviation 3.7 minutes.

(i) For a random sample of 350 students, find the number who would be expected to take longer than 20 minutes to get up in the morning. [3]

(ii) 'Very slow' students are students whose time to get up is more than 1.645 standard deviations above the mean. Find the probability that fewer than 3 students from a random sample of 8 students are 'very slow'. [4]

Answer: i) 335 ii) 0.994 **61/N10/3**

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34 The heights that children of a particular age can jump have a normal distribution. On average, 8 children out of 10 can jump a height of more than 127 cm, and 1 child out of 3 can jump a height of more than 135 cm.

(i) Find the mean and standard deviation of the heights the children can jump. [5]

(ii) Find the probability that a randomly chosen child will not be able to jump a height of 145 cm. [3]

(iii) Find the probability that, of 8 randomly chosen children, at least 2 will be able to jump a height of more than 135 cm. [3]

Answer: i) 132, 6.29 ii) 0.978 iii) 0.805 **63/J10**

---

35 The random variable X is the length of time in minutes that Jannon takes to mend a bicycle puncture. X has a normal distribution with mean μ and variance σ^2. It is given that $P(X > 30.0) = 0.1480$ and $P(X > 20.9) = 0.6228$. Find μ and σ. [5]

Answer: 23.0, 6.70 **61/J10/3**

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36 The height of maize plants in Mpapwa is normally distributed with mean 1.62 m and standard deviation σ m. The probability that a randomly chosen plant has a height greater than 1.8 m is 0.15. Find the value of σ. [3]

Answer: 0.174 **J16/61/Q1**

37 The heights of school desks have a normal distribution with mean 69 cm and standard deviation σ cm. It is known that 15.5% of these desks have a height greater than 70 cm.

(i) Find the value of σ. [3]

When Jodu sits at a desk, his knees are at a height of 58 cm above the floor. A desk is comfortable for Jodu if his knees are at least 9 cm below the top of the desk. Jodu's school has 300 desks.

(ii) Calculate an estimate of the number of these desks that are comfortable for Jodu. [5]

Answers: (i) 0.985 (ii) 293 or 294 **J16/63/Q5**

38 **(a)** The random variable X has a normal distribution with mean μ and standard deviation σ. You are given that $\sigma = 0.25\mu$ and $P(X < 6.8) = 0.75$.

(i) Find the value of μ. [4]

(ii) Find $P(X < 4.7)$. [3]

(b) The lengths of metal rods have a normal distribution with mean 16 cm and standard deviation 0.2 cm. Rods which are shorter than 15.75 cm or longer than 16.25 cm are not usable. Find the expected number of usable rods in a batch of 1000 rods. [4]

Answer: (a)(i) 5.82 (ii) 0.221 (b) 788 or 789

J17/61/Q6

39 **(a)** The random variable X has the distribution $N(\mu, \sigma^2)$, where $\mu = 1.5\sigma$. A random value of X is chosen. Find the probability that this value of X is greater than 0. [3]

(b) The life of a particular type of torch battery is normally distributed with mean 120 hours and standard deviation s hours. It is known that 87.5% of these batteries last longer than 70 hours. Find the value of s. [3]

Answers: (a) 0.933 (b) 43.4 or 43.5

J17/63/Q4

40 The random variable X is such that $X \sim N(20, 49)$. Given that $P(X > k) = 0.25$, find the value of k. [3]

Answer: 24.7

N16/61/Q1

41 Packets of rice are filled by a machine and have weights which are normally distributed with mean 1.04 kg and standard deviation 0.017 kg.

(i) Find the probability that a randomly chosen packet weighs less than 1 kg. [3]

(ii) How many packets of rice, on average, would the machine fill from 1000 kg of rice? [1]

The factory manager wants to produce more packets of rice. He changes the settings on the machine so that the standard deviation is the same but the mean is reduced to μ kg. With this mean the probability that a packet weighs less than 1 kg is 0.0388.

(iii) Find the value of μ. [3]

(iv) How many packets of rice, on average, would the machine now fill from 1000 kg of rice? [1]

Answers (i) 0.0093 (ii) 961 or 962 (iii) 1.03 (iv) 971 or 970

N16/61/Q4

42 The weights of bananas in a fruit shop have a normal distribution with mean 150 grams and standard deviation 50 grams. Three sizes of banana are sold.

Small: under 95 grams
Medium: between 95 grams and 205 grams
Large: over 205 grams

(i) Find the proportion of bananas that are small. [3]

(ii) Find the weight exceeded by 10% of bananas. [3]

The prices of bananas are 10 cents for a small banana, 20 cents for a medium banana and 25 cents for a large banana.

(iii) (a) Show that the probability that a randomly chosen banana costs 20 cents is 0.7286. [1]

(b) Calculate the expected total cost of 100 randomly chosen bananas. [3]

N16/63/Q6

43 The weight, in grams, of pineapples is denoted by the random variable X which has a normal distribution with mean 500 and standard deviation 91.5. Pineapples weighing over 570 grams are classified as 'large'. Those weighing under 390 grams are classified as 'small' and the rest are classified as 'medium'.

(i) Find the proportions of large, small and medium pineapples. [5]

(ii) Find the weight exceeded by the heaviest 5% of pineapples. [3]

(iii) Find the value of k such that $P(k < X < 610) = 0.3$. [5]

Answers: (i) Large: 0.222, Medium: 0.663, Small: 0.115 (ii) 651 (iii) 520 **N17/61/Q7**

44 Josie aims to catch a bus which departs at a fixed time every day. Josie arrives at the bus stop T minutes before the bus departs, where $T \sim N(5.3, 2.1^2)$.

(i) Find the probability that Josie has to wait longer than 6 minutes at the bus stop. [3]

On 5% of days Josie has to wait longer than x minutes at the bus stop.

(ii) Find the value of x. [3]

(iii) Find the probability that Josie waits longer than x minutes on fewer than 3 days in 10 days. [3]

(iv) Find the probability that Josie misses the bus. [3]

Answers:(i) 0.370, (ii) 8.75, (iii) 0.988, (iv) 0.0058 **N17/63/Q7**

45 **(a)** It is given that $X \sim N(31.4, 3.6)$. Find the probability that a randomly chosen value of X is less than 29.4. [3]

(b) The lengths of fish of a particular species are modelled by a normal distribution. A scientist measures the lengths of 400 randomly chosen fish of this species. He finds that 42 fish are less than 12 cm long and 58 are more than 19 cm long. Find estimates for the mean and standard deviation of the lengths of fish of this species. **[5]**

Answers: (i) 0.146 (ii) $\mu = 15.8$, $\sigma = 3.03$

N18/61/Q4

46 The weights of apples sold by a store can be modelled by a normal distribution with mean 120 grams and standard deviation 24 grams. Apples weighing less than 90 grams are graded as 'small'; apples weighing more than 140 grams are graded as 'large'; the remainder are graded as 'medium'.

(i) Show that the probability that an apple chosen at random is graded as medium is 0.692, correct to 3 significant figures. [4]

(ii) Four apples are chosen at random. Find the probability that at least two are graded as medium. [4]

Answer: 0.692 AG *Answer:* 0.910

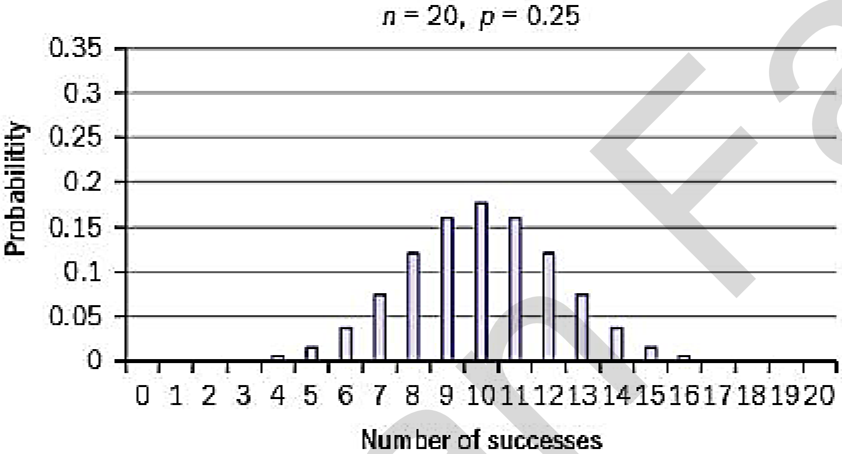
N18/63/Q5

THE NORMAL APPROXIMATION TO THE BINOMIAL DISTRIBUTION

Discrete **random variables** take only particular values, each with its own **probability**. *Continuous* random variables take values over an interval, and probabilities are defined for **ranges** of values rather than individual values.

When there are a large number of possible values for a discrete distribution there can be a lot of calculations involved.

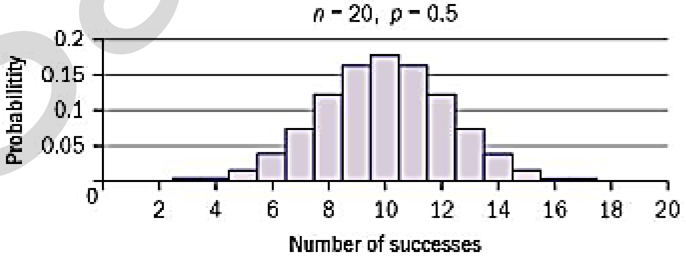
However, if we consider the graphs of some of the distributions, their shape looks similar to the **normal distribution** and suggests that we could use the corresponding normal distribution to reduce the work in calculating probabilities. There is a trade-off between only getting an approximation to the probability and doing much less work.



CONTINUITY CORRECTIONS

In the diagrams in Section 10.1 the discrete probabilities are shown as bars with gaps between them. In fact, the bars really should have zero width since the probability occurs only at the integer value, in which case the diagrams would not look so much like the normal.

If we take '7' in the discrete distribution to be represented by the interval (6.5, 7.5), which are the values which round to 7, then the graphs look like:



The resemblance to the normal distribution is now even stronger.

When the normal is used to approximate the binomial (or any other distribution that takes only integer values) we must use a **continuity correction.**

Example 7.1

Let $X \sim B(n, p)$ and $Y \sim N(np, npq)$, where n, p satisfy the conditions needed for Y to be used as an approximation for X, and $q = 1 - p$.

Write down the probability you need to calculate for Y (including the continuity correction) as the approximation for each of the following probabilities for X.

a) $P(X < 15)$ **b)** $P(X > 12)$ **c)** $P(X \leq 17)$ **d)** $P(12 < X \leq 15)$.

Exercise 7e Continuity corrections

Write down the transformations for each of the following, when a normal distribution is to be used as an approximation for a binomial distribution.

1. $P(3 \leqslant X \leqslant 9)$
2. $P(3 < X < 9)$
3. $P(10 < X \leqslant 24)$
4. $P(2 \leqslant X < 8)$
5. $P(X > 54)$
6. $P(X \geqslant 76)$
7. $P(45 < X < 67)$
8. $P(X < 109)$
9. $P(X \leqslant 45)$
10. $P(X = 56)$
11. $P(400 < X \leqslant 560)$
12. $P(X = 67)$
13. $P(X > 59)$
14. $P(X = 100)$
15. $P(34 \leqslant X < 43)$
16. $P(X = 7)$
17. $P(X \geqslant 509)$
18. $P(X < 7)$
19. $P(27 \leqslant X < 29)$
20. $P(X = 53)$

Exercise 7e Continuity corrections (page 386)

1. $P(2.5 < X < 9.5)$
2. $P(3.5 < X < 8.5)$
3. $P(10.5 < X < 24.5)$
4. $P(1.5 < X < 7.5)$
5. $P(X > 54.5)$
6. $P(X > 75.5)$
7. $P(45.5 < X < 66.5)$
8. $P(X < 108.5)$
9. $P(X < 45.5)$
10. $P(55.5 < X < 56.5)$
11. $P(400.5 < X < 560.5)$
12. $P(66.5 < X < 67.5)$
13. $P(X > 59.5)$
14. $P(99.5 < X < 100.5)$
15. $P(33.5 < X < 42.5)$
16. $P(6.5 < X < 7.5)$
17. $P(X > 508.5)$
18. $P(X < 6.5)$
19. $P(26.5 < X < 28.5)$
20. $P(52.5 < X < 53.5)$

THE PARAMETERS FOR THE NORMAL APPROXIMATION

In Section 7.2 we saw that if $X \sim B(n, p)$, then $E(X) = np$ and $\text{Var}(X) = npq = np(1-p)$.

If n is large and p is close to 0.5, so that the distribution is nearly **symmetric**, then you can use the normal distribution to approximate the binomial.

The **parameters** to be used are the **mean** and **variance** of the binomial: $\mu = np$; $\sigma^2 = npq$.

The normal distribution is symmetric, and so is the binomial when $p = 0.5$, but as n gets larger the requirement for p to be close to 0.5 becomes less important.

The general rule is that the normal distribution can be used as an approximation when both np and nq are > 5.

Example 7.2

In a sack of mixed grass seeds, the probability that a seed is ryegrass is 0.35.
Find the probability that in a random sample of 400 seeds from the sack,
(a) Less than 120 are ryegrass seeds,
(b) Between 120 and 150 (inclusive) are ryegrass,
(c) More than 160 are ryegrass seeds.

Example 7.3

It is given that 40% of the population support the Gamboge Party. One hundred and fifty member of the population are selected at random. Use a suitable approximation to find the probability that more than 55 out of the 150 support the Gamboge Party.

Example 7.4

An airline estimates that 7% of passengers who book seats on a flight are 'no shows' – for one reason or another they miss the flight. On one flight, for which there are 185 available seats, the airline sells 197 tickets.

Find the probability that the airline will not have to refuse boarding to any passengers holding a valid ticket for that flight.

Example 7.5

It is known that, on average, 2 people in 5 in a certain country are overweight.
A random sample of 400 people is chosen. Using a suitable approximation, find the probability that fewer than 165 people in the sample are overweight.

[Cambridge International AS and A Level Mathematics 9709, Paper 6 Q1 June 2005]

Example 7.6

A survey of adults in a certain large town found that 76% of people wore a watch on their left wrist, 15% wore a watch on their right wrist and 9% did not wear a watch.

(i) A random sample of 14 adults was taken. Find the probability that more than 2 adults did not wear a watch.

(ii) A random sample of 200 adults was taken. Using a suitable approximation, find the probability that more than 155 wore a watch on their left wrist.

[Cambridge International AS and A Level Mathematics 9709, Paper 6 Q7 June 2006]

Example 7.7

On a certain road 20% of the vehicles are trucks, 16% are buses and the remainder are cars.

(i) A random sample of 11 vehicles is taken. Find the probability that fewer than 3 are buses.

(ii) A random sample of 125 vehicles is now taken. Using a suitable approximation, find the probability that more than 73 are cars.

[Cambridge International AS and A Level Mathematics 9709, Paper 6 Q3 June 2009]

Example 7.8

On any occasion when a particular gymnast performs a certain routine, the probability that she will perform it correctly is 0.65, independently of all other occasions.

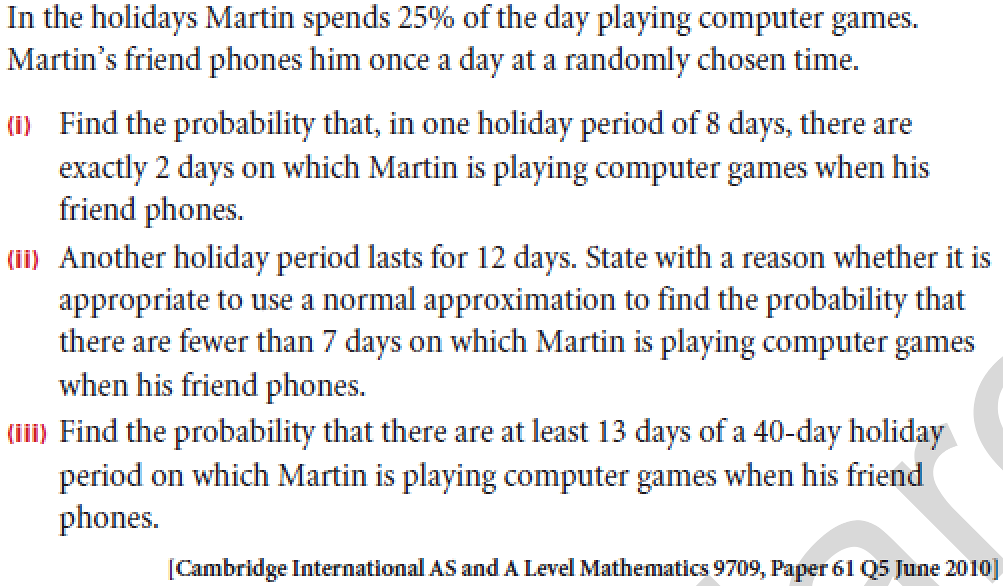
(i) Find the probability that she will perform the routine correctly on exactly 5 occasions out of 7.

(ii) On one day she performs the routine 50 times. Use a suitable approximation to estimate the probability that she will perform the routine correctly on fewer than 29 occasions.

(iii) On another day she performs the routine n times. Find the smallest value of n for which the expected number of correct performances is at least 8.

[Cambridge International AS and A Level Mathematics 9709, Paper 6 Q6 November 2007]

Example 7.9



In the holidays Martin spends 25% of the day playing computer games.
Martin's friend phones him once a day at a randomly chosen time.

(i) Find the probability that, in one holiday period of 8 days, there are exactly 2 days on which Martin is playing computer games when his friend phones.

(ii) Another holiday period lasts for 12 days. State with a reason whether it is appropriate to use a normal approximation to find the probability that there are fewer than 7 days on which Martin is playing computer games when his friend phones.

(iii) Find the probability that there are at least 13 days of a 40-day holiday period on which Martin is playing computer games when his friend phones.

[Cambridge International AS and A Level Mathematics 9709, Paper 61 Q5 June 2010]

<u>MISCELLANEOUS QUESTIONS</u>

Example 7.5

A product is sold in packets whose masses are normally distributed with a mean of 1.42 kg and a standard deviation of 0.025kg.

(a) Find the probability that the mass of a packet, selected at random, lies between 1.37 kg and 1.45 kg.

(b) Estimate the number of packets in an output of 5000, whose mass is less than 1.35kg.

Example 6.4

Machine A, used for filling bags with ground coffee, can be set to dispense any required mean weight of coffee per bag. At any setting the weight of coffee in a bag can be modelled by a normal distribution with a standard deviation of 1.95 g.

(a) If the machine is set to dispense a mean weight of 128 g of coffee per bag, calculate the percentage of bag that contain less than 125 g.

(b) To meet an official regulation the setting on a machine must be adjusted so that no more than 1% of bags contain less than 125 g.

(i) Calculate the smallest mean weight to which machine A should be set to meet the regulation.

(ii) Machine B will only just meet the regulation when it is set to dispense a mean weight of 128.5 g. Assuming that the weight of coffee is a bag filled by Machine B can be modeled by a normal distribution, calculate the standard deviation of this distribution.

Example 6.5

It is estimated that, on a average, one match in five in the Football League is drawn, and that one match in two is a home win.

(a) Twelve matches are selected at random. Calculate the probability that the number of drawn matches is,

(i) Exactly three,

(ii) At least four.

(b) Ninety matches are selected at random. Use a suitable approximation to calculate the probability that between 13 and 20 (inclusive) of the matches are drawn.

(c) Twenty matches are selected at random. The random variable D and H are the numbers of drawn matches and home wins, respectively, in these matches. State, with a reason, which of D and H can be better approximated by a normal variable.

HOMEWORK: CONVERSION OF DISTRIBUTIONS– VARIANT 62

1 Kamal has 30 hens. The probability that any hen lays an egg on any day is 0.7. Hens do not lay more than one egg per day, and the days on which a hen lays an egg are independent.

(i) Calculate the probability that, on any particular day, Kamal's hens lay exactly 24 eggs. [2]

(ii) Use a suitable approximation to calculate the probability that Kamal's hens lay fewer than 20 eggs on any particular day. [5]

Answers: (i) 0.0829; (ii) 0.275 . **J03/Q4**

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2 A shop sells old video tapes, of which 1 in 5 on average are known to be damaged.

(i) A random sample of 15 tapes is taken. Find the probability that at most 2 are damaged. [3]

(ii) Find the smallest value of n if there is a probability of at least 0.85 that a random sample of n tapes contains at least one damaged tape. [3]

(iii) A random sample of 1600 tapes is taken. Use a suitable approximation to find the probability that there are at least 290 damaged tapes. [5]

Answers: (i) 0.398; (ii) 9; (iii) 0.972. **J04/Q7**

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3 It is known that, on average, 2 people in 5 in a certain country are overweight. A random sample of 400 people is chosen. Using a suitable approximation, find the probability that fewer than 165 people in the sample are overweight. [5]

Answers: (i) 0.398; (ii) 9; (iii) 0.972. **J05/Q1**

---

4 A survey of adults in a certain large town found that 76% of people wore a watch on their left wrist, 15% wore a watch on their right wrist and 9% did not wear a watch.

(i) A random sample of 14 adults was taken. Find the probability that more than 2 adults did not wear a watch. [4]

(ii) A random sample of 200 adults was taken. Using a suitable approximation, find the probability that more than 155 wore a watch on their left wrist. [5]

Answers: (i) 0.126; (ii) 0.281. **J06/Q7**

5 The probability that New Year's Day is on a Saturday in a randomly chosen year is $\frac{1}{7}$.

(i) 15 years are chosen randomly. Find the probability that at least 3 of these years have New Year's Day on a Saturday. [4]

(ii) 56 years are chosen randomly. Use a suitable approximation to find the probability that more than 7 of these years have New Year's Day on a Saturday. [5]

Answers: (i) 0.365; (ii) 0.576. **J07/Q6**

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6 A die is biased so that the probability of throwing a 5 is 0.75 and the probabilities of throwing a 1, 2, 3, 4 or 6 are all equal.

(i) The die is thrown three times. Find the probability that the result is a 1 followed by a 5 followed by any even number. [3]

(ii) Find the probability that, out of 10 throws of this die, at least 8 throws result in a 5. [3]

(iii) The die is thrown 90 times. Using an appropriate approximation, find the probability that a 5 is thrown more than 60 times. [5]

Answers: (i) 0.00563; (ii) 0.526; (iii) 0.956. **J08/Q7**

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7 On a certain road 20% of the vehicles are trucks, 16% are buses and the remainder are cars.

(i) A random sample of 11 vehicles is taken. Find the probability that fewer than 3 are buses. [3]

(ii) A random sample of 125 vehicles is now taken. Using a suitable approximation, find the probability that more than 73 are cars. [5]

Answers: (i) 0.748; (ii) 0.887. **J09/Q3**

---

8 **(i)** A manufacturer of biscuits produces 3 times as many cream ones as chocolate ones. Biscuits are chosen randomly and packed into boxes of 10. Find the probability that a box contains equal numbers of cream biscuits and chocolate biscuits. [2]

(ii) A random sample of 8 boxes is taken. Find the probability that exactly 1 of them contains equal numbers of cream biscuits and chocolate biscuits. [2]

(iii) A large box of randomly chosen biscuits contains 120 biscuits. Using a suitable approximation, find the probability that it contains fewer than 35 chocolate biscuits. [5]

Answers: (i) 0.0584; (ii) 0.307; (iii) 0.829. **N02/Q6**

9 **(i)** State two conditions which must be satisfied for a situation to be modelled by a binomial distribution. [2]

In a certain village 28% of all cars are made by Ford.

(ii) 14 cars are chosen randomly in this village. Find the probability that fewer than 4 of these cars are made by Ford. [4]

(iii) A random sample of 50 cars in the village is taken. Estimate, using a normal approximation, the probability that more than 18 cars are made by Ford. [4]

Answers: (ii) 0.419; (iii) 0.0782. **N04/Q7**

---

10 In tests on a new type of light bulb it was found that the time they lasted followed a normal distribution with standard deviation 40.6 hours. 10% lasted longer than 5130 hours.

(i) Find the mean lifetime, giving your answer to the nearest hour. [3]

(ii) Find the probability that a light bulb fails to last for 5000 hours. [3]

(iii) A hospital buys 600 of these light bulbs. Using a suitable approximation, find the probability that fewer than 65 light bulbs will last longer than 5130 hours. [4]

Answers: (i) 5080; (ii) 0.0273; (iii) 0.730. **N05/Q7**

---

11 A manufacturer makes two sizes of elastic bands: large and small. 40% of the bands produced are large bands and 60% are small bands. Assuming that each pack of these elastic bands contains a random selection, calculate the probability that, in a pack containing 20 bands, there are

(i) equal numbers of large and small bands, [2]

(ii) more than 17 small bands. [3]

An office pack contains 150 elastic bands.

(iii) Using a suitable approximation, calculate the probability that the number of small bands in the office pack is between 88 and 97 inclusive. [6]

Answers: (i) 0.117; (ii) 0.00361; (iii) 0.556. **N06/Q7**

12 On any occasion when a particular gymnast performs a certain routine, the probability that she will perform it correctly is 0.65, independently of all other occasions.

(i) Find the probability that she will perform the routine correctly on exactly 5 occasions out of 7. [2]

(ii) On one day she performs the routine 50 times. Use a suitable approximation to estimate the probability that she will perform the routine correctly on fewer than 29 occasions. [5]

(iii) On another day she performs the routine n times. Find the smallest value of n for which the expected number of correct performances is at least 8. [2]

Answers: (i) 0.298; (ii) 0.118; (iii) 13. **N07/Q6**

13 On a production line making toys, the probability of any toy being faulty is 0.08. A random sample of 200 toys is checked. Use a suitable approximation to find the probability that there are at least 15 faulty toys. [5]

N08/Q2

Answer: 0.652.

14

(i) State three conditions that must be satisfied for a situation to be modelled by a binomial distribution. [2]

On any day, there is a probability of 0.3 that Julie's train is late.

(ii) Nine days are chosen at random. Find the probability that Julie's train is late on more than 7 days or fewer than 2 days. [3]

(iii) 90 days are chosen at random. Find the probability that Julie's train is late on more than 35 days or fewer than 27 days. [5]

Answers: (ii) 0.196; (iii) 0.480. **N10/62/Q6**

15 In Scotland, in November, on average 80% of days are cloudy. Assume that the weather on any one day is independent of the weather on other days.

(i) Use a normal approximation to find the probability of there being fewer than 25 cloudy days in Scotland in November (30 days). [4]

(ii) Give a reason why the use of a normal approximation is justified. [1]

Answer: (i) 0.590. **J11/62/Q2**

16 The time Rafa spends on his homework each day in term-time has a normal distribution with mean 1.9 hours and standard deviation σ hours. On 80% of these days he spends more than 1.35 hours on his homework.

(i) Find the value of σ. [3]

(ii) Find the probability that, on a randomly chosen day in term-time, Rafa spends less than 2 hours on his homework. [2]

(iii) A random sample of 200 days in term-time is taken. Use an approximation to find the probability that the number of days on which Rafa spends more than 1.35 hours on his homework is between 163 and 173 inclusive. [6]

Answers: 0.653, 0.561, 0.321 **J14/62/Q7**

---

17 A triangular spinner has one red side, one blue side and one green side. The red side is weighted so that the spinner is four times more likely to land on the red side than on the blue side. The green side is weighted so that the spinner is three times more likely to land on the green side than on the blue side.

(i) Show that the probability that the spinner lands on the blue side is $\frac{1}{8}$. [1]

(ii) The spinner is spun 3 times. Find the probability that it lands on a different coloured side each time. [3]

(iii) The spinner is spun 136 times. Use a suitable approximation to find the probability that it lands on the blue side fewer than 20 times. [5]

Answers: (ii) 9/64 (0.141); (iii) 0.742 **N11/62/Q5**

---

18 The mean of a certain normally distributed variable is four times the standard deviation. The probability that a randomly chosen value is greater than 5 is 0.15.

(i) Find the mean and standard deviation. [4]

(ii) 200 values of the variable are chosen at random. Find the probability that at least 160 of these values are less than 5. [5]

Answers: (i) 0.993, 3.97; (ii) 0.981. **N12/62/Q4**

---

19 On trains in the morning rush hour, each person is either a student with probability 0.36, or an office worker with probability 0.22, or a shop assistant with probability 0.29 or none of these.

(i) 8 people on a morning rush hour train are chosen at random. Find the probability that between 4 and 6 inclusive are office workers. [3]

(ii) 300 people on a morning rush hour train are chosen at random. Find the probability that between 31 and 49 inclusive are neither students nor office workers nor shop assistants. [6]

Answers: 0.0763, 0.892 **N13/62/Q5**

20 In Marumbo, three quarters of the adults own a cell phone.

(i) A random sample of 8 adults from Marumbo is taken. Find the probability that the number of adults who own a cell phone is between 4 and 6 inclusive. [3]

(ii) A random sample of 160 adults from Marumbo is taken. Use an approximation to find the probability that more than 114 of them own a cell phone. [5]

(iii) Justify the use of your approximation in part **(ii)**. [1]

Answers: 0.606, 0.842, $np > 5$ and $nq > 5$ **N14/62/Q7**

HOMEWORK: CONVERSION OF DISTRIBUTIONS VARIANT 61 AND 63

1 (i) In a certain country, 68% of households have a printer. Find the probability that, in a random sample of 8 households, 5, 6 or 7 households have a printer. [4]

(ii) Use an approximation to find the probability that, in a random sample of 500 households, more than 337 households have a printer. [5]

(iii) Justify your use of the approximation in part **(ii)**. [1]

Answer: i) 0.722 ii) 0.595 iii) np>5, nq>5 **61/J15/6**

---

2 On a production line making cameras, the probability of a randomly chosen camera being substandard is 0.072. A random sample of 300 cameras is checked. Find the probability that there are fewer than 18 cameras which are substandard. [5]

Answer: 0.180 **63/J15/3**

---

3 A factory makes water pistols, 8% of which do not work properly.

(i) A random sample of 19 water pistols is taken. Find the probability that at most 2 do not work properly. [3]

(ii) In a random sample of n water pistols, the probability that at least one does not work properly is greater than 0.9. Find the smallest possible value of n. [3]

(iii) A random sample of 1800 water pistols is taken. Use an approximation to find the probability that there are at least 152 that do not work properly. [5]

(iv) Justify the use of your approximation in part **(iii)**. [1]

Answer: i) 0.809 ii) 28 iii)0.257 iv) np>5, nq>5 **63/N15/7**

---

4 In Marumbo, three quarters of the adults own a cell phone.

(i) A random sample of 8 adults from Marumbo is taken. Find the probability that the number of adults who own a cell phone is between 4 and 6 inclusive. [3]

(ii) A random sample of 160 adults from Marumbo is taken. Use an approximation to find the probability that more than 114 of them own a cell phone. [5]

(iii) Justify the use of your approximation in part **(ii)**. [1]

Answer: i) 0.606 ii) 0.842 iii) np>5 nq>5 **62/N14/7**

5 There is a probability of $\frac{1}{7}$ that Wenjie goes out with her friends on any particular day. 252 days are chosen at random.

(i) Use a normal approximation to find the probability that the number of days on which Wenjie goes out with her friends is less than than 30 or more than 44. [5]

(ii) Give a reason why the use of a normal approximation is justified. [1]

Answer: i) 0.184 ii) np>5 and nq>5 **63/J14/2**

---

6 Lengths of a certain type of carrot have a normal distribution with mean 14.2 cm and standard deviation 3.6 cm.

(i) 8% of carrots are shorter than c cm. Find the value of c. [3]

(ii) Rebekah picks 7 carrots at random. Find the probability that at least 2 of them have lengths between 15 and 16 cm. [6]

Answer: i) 9.14 ii) 0.159 **61/N13/5**

---

7 In a certain country, on average one student in five has blue eyes.

(i) For a random selection of n students, the probability that none of the students has blue eyes is less than 0.001. Find the least possible value of n. [3]

(ii) For a random selection of 120 students, find the probability that fewer than 33 have blue eyes. [4]

Answer: i) 31 ii) 0.974 **63/J13/4**

---

8 Assume that, for a randomly chosen person, their next birthday is equally likely to occur on any day of the week, independently of any other person's birthday. Find the probability that, out of 350 randomly chosen people, at least 47 will have their next birthday on a Monday. [5]

Answer: 0.704 **61/J13/2**

---

9 Ana meets her friends once every day. For each day the probability that she is early is 0.05 and the probability that she is late is 0.75. Otherwise she is on time.

(i) Find the probability that she is on time on fewer than 20 of the next 96 days. [5]

(ii) If she is early there is a probability of 0.7 that she will eat a banana. If she is late she does not eat a banana. If she is on time there is a probability of 0.4 that she will eat a banana. Given that for one particular meeting with friends she does not eat a banana, find the probability that she is on time. [4]

Answer: i) 0.531 ii) 0.136 **61/N12/6**

---

10 Human blood groups are identified by two parts. The first part is A, B, AB or O and the second part (the Rhesus part) is + or –. In the UK, 35% of the population are group A+, 8% are B+, 3% are AB+, 37% are O+, 7% are A–, 2% are B–, 1% are AB– and 7% are O–.

(i) A random sample of 9 people in the UK who are Rhesus + is taken. Find the probability that fewer than 3 are group O+. [6]

(ii) A random sample of 150 people in the UK is taken. Find the probability that more than 60 people are group A+. [5]

Answer: i) 0.156 ii) 0.0854 **63/N11/6**

---

11 When a butternut squash seed is sown the probability that it will germinate is 0.86, independently of any other seeds. A market gardener sows 250 of these seeds. Use a suitable approximation to find the probability that more than 210 germinate. [5]

Answer: 0.794 **61/N11/1**

---

12 The times spent by people visiting a certain dentist are independent and normally distributed with a mean of 8.2 minutes. 79% of people who visit this dentist have visits lasting less than 10 minutes.

(i) Find the standard deviation of the times spent by people visiting this dentist. [3]

(ii) Find the probability that the time spent visiting this dentist by a randomly chosen person deviates from the mean by more than 1 minute. [3]

(iii) Find the probability that, of 6 randomly chosen people, more than 2 have visits lasting longer than 10 minutes. [3]

(iv) Find the probability that, of 35 randomly chosen people, fewer than 16 have visits lasting less than 8.2 minutes. [5]

Answer: i) 2.23 ii) 0.654 iii) 0.112 iv) 0.250 **63/N10/7**

13 In the holidays Martin spends 25% of the day playing computer games. Martin's friend phones him once a day at a randomly chosen time.

(i) Find the probability that, in one holiday period of 8 days, there are exactly 2 days on which Martin is playing computer games when his friend phones. [2]

(ii) Another holiday period lasts for 12 days. State with a reason whether it is appropriate to use a normal approximation to find the probability that there are fewer than 7 days on which Martin is playing computer games when his friend phones. [1]

(iii) Find the probability that there are at least 13 days of a 40-day holiday period on which Martin is playing computer games when his friend phones. [5]

Answer: i) 0.311 iii) 0.181 **61/J10/5**

---

14 Plastic drinking straws are manufactured to fit into drinks cartons which have a hole in the top. A straw fits into the hole if the diameter of the straw is less than 3 mm. The diameters of the straws have a normal distribution with mean 2.6 mm and standard deviation 0.25 mm.

(i) A straw is chosen at random. Find the probability that it fits into the hole in a drinks carton. [3]

(ii) 500 straws are chosen at random. Use a suitable approximation to find the probability that at least 480 straws fit into the holes in drinks cartons. [5]

(iii) Justify the use of your approximation. [1]

Answers: **(i)** 0.945 **(ii)** 0.0875 **(iii)** $np > 5$, $nq > 5$ **J16/61/Q5**

---

15 Passengers are travelling to Picton by minibus. The probability that each passenger carries a backpack is 0.65, independently of other passengers. Each minibus has seats for 12 passengers.

(i) Find the probability that, in a full minibus travelling to Picton, between 8 passengers and 10 passengers inclusive carry a backpack. [3]

(ii) Passengers get on to an empty minibus. Find the probability that the fourth passenger who gets on to the minibus will be the first to be carrying a backpack. [2]

(iii) Find the probability that, of a random sample of 250 full minibuses travelling to Picton, more than 54 will contain exactly 7 passengers carrying backpacks. [6]

Answer: **(i)** 0.541 **(ii)** 0.0279 **(iii)** 0.290 **J16/63/Q7**

---

16 The probability that George goes swimming on any day is $\frac{1}{3}$. Use an approximation to calculate the probability that in 270 days George goes swimming at least 100 times. [5]

Answer: 0.110

J17/63/Q2

---

17 Each day Annabel eats rice, potato or pasta. Independently of each other, the probability that she eats rice is 0.75, the probability that she eats potato is 0.15 and the probability that she eats pasta is 0.1.

(i) Find the probability that, in any week of 7 days, Annabel eats pasta on exactly 2 days. [2]

(ii) Find the probability that, in a period of 5 days, Annabel eats rice on 2 days, potato on 1 day and pasta on 2 days. [3]

(iii) Find the probability that Annabel eats potato on more than 44 days in a year of 365 days. [5]

Answers: (i) 0.124 (ii) 0.0253 (iii) 0.933 (or 0.944)

N16/63/Q7

---

18 At the Nonland Business College, all students sit an accountancy examination at the end of their first year of study. On average, 80% of the students pass this examination.

(i) A random sample of 9 students who will take this examination is chosen. Find the probability that at most 6 of these students will pass the examination. [3]

(ii) A random sample of 200 students who will take this examination is chosen. Use a suitable approximate distribution to find the probability that more than 166 of them will pass the examination. [5]

Answers: (ii) 0.262 (ii) 0.125 (iii) $np = 160$ $nq = 40$, both > 5 so normal approximation holds

N18/61/Q5

---

19 The lifetimes, in hours, of a particular type of light bulb are normally distributed with mean 2000 hours and standard deviation σ hours. The probability that a randomly chosen light bulb of this type has a lifetime of more than 1800 hours is 0.96.

(i) Find the value of σ. [3]

New technology has resulted in a new type of light bulb. It is found that on average one in five of these new light bulbs has a lifetime of more than 2500 hours.

(ii) For a random selection of 300 of these new light bulbs, use a suitable approximate distribution to find the probability that fewer than 70 have a lifetime of more than 2500 hours. [4]

(iii) Justify the use of your approximate distribution in part **(ii)**. [1]

Answer: 114 *Answer:* 0.915 *Answer:* np = 60 > 5; nq = 240 > 5

N18/63/Q6

PERMUTATIONS AND COMBINATIONS

COMBINATIONS

1. A school council of 6 people is to be chosen from a group of 8 students and 6 teachers. Calculate the number of different ways that the council can be selected if

(i) there are no restrictions, **3003** [2]
(ii) there must be at least 1 teacher on the council and more students than teachers. **1386** [3]

2. A committee of 4 people is to be chosen from 4 women and 5 men. The committee must contain at least 1 woman. Calculate the number of different committees that can be formed.

3. A committee of 5 people is to be selected from 6 men and 4 women.
Find
   (i) the number of different ways in which the committee can be selected, **252** [1]
   (ii) the number of these selections with more women than men. **66**

4. A music student needs to select 7 pieces of music from 6 classical pieces and 4 modern pieces. Find the number of different selections that she can make if
   a. there are no restrictions, **120** [1]
   b. there are to be only 2 modern pieces included, **36** [2]
   c. there are to be more classical pieces than modern pieces. **100** [4]

5. An examination paper contains 12 different questions of which 3 are on trigonometry, 4 are on algebra and 5 are on calculus. Candidates are asked to answer 8 questions. Calculate
   a. the number of different ways in which a candidate can select 8 questions if there is no restriction, **495**
   b. the number of these selections which contain questions on only 2 of the 3 topics, trigonometry, algebra and calculus. **10**

6. There are 7 Chinese, 6 European and 4 American students at an international conference. Four of the students are to be chosen to take part in a television broadcast. Find the number of different ways the students can be chosen if at least one Chinese and at least one European student are included. **1841** [5]

7. A committee of 6 people is to be chosen from 5 men and 8 women. In how many ways can this be done
   (i) if there are more women than men on the committee, **1008** [4]
   (ii) if the committee consists of 3 men and 3 women but two particular men refuse to be on the committee together? **392** [3]

PERMUTATIONS OF LETTERS

1.

(i) Find the number of different arrangements of the letters of the word MEXICO. 720

Find the number of these arrangements

(ii) which begin with M, **120**

(iii) which have the letter X at one end and the letter C at the other end. **48**

Four of the letters of the word MEXICO are selected at random. Find the number of different combinations if

(iv) there is no restriction on the letters selected, **15**

(v) the letter M must be selected. **10**

2. .

(a) Find the number of ways in which all nine letters of the word TENNESSEE can be arranged

(i) if all the letters E are together, [3] **180**

(ii) if the T is at one end and there is an S at the other end. [3] **210**

(b) Four letters are selected from the nine letters of the word VENEZUELA. Find the number of possible selections which contain exactly one E. [3] **20**

3. Find the number of different ways that the 13 letters of the word ACCOMMODATION can be arranged in a line if all the vowels (A, I, O) are next to each other. [3] **604800**

4. The 11 letters of the word REMEMBRANCE are arranged in a line.

(i) Find the number of different arrangements if there are no restrictions. [1] **1663200**

(ii) Find the number of different arrangements which start and finish with the letter M. [2] **30240**

(iii) Find the number of different arrangements which do not have all 4 vowels (E, E, A, E) next to each other. [3] **1622880**

(iv) 4 letters from the letters of the word REMEMBRANCE are chosen. Find the number of different selections which contain no Ms and no Rs and at least 2 Es. [3] **10**

5.

(i) Find the number of different ways that the 9 letters of the word AGGREGATE can be arranged in a line if the first letter is R. [2] **1680**

(ii) Find the number of different ways that the 9 letters of the word AGGREGATE can be arranged in a line if the 3 letters G are together, both letters A are together and both letters E are together. [2] **120**

(iii) The letters G, R and T are consonants and the letters A and E are vowels. Find the number of different ways that the 9 letters of the word AGGREGATE can be arranged in a line if consonants and vowels occur alternately. [3] **120**

(iv) Find the number of different selections of 4 letters of the word AGGREGATE which contain exactly 2 Gs or exactly 3 Gs. [3] **12**

PERMUTATIONS OF DIGITS

1. A 4-digit number is formed by using four of the seven digits 1, 3, 4, 5, 7, 8 and 9. No digit can be used more than once in any one number. Find how many different 4-digit numbers can be formed if

(i) there are no restrictions, [2] **840**
(ii) the number is less than 4000, [2] **240**
(iii) the number is even and less than 4000. [2] **80**

2. A 4-digit number is formed by using four of the seven digits 2, 3, 4, 5, 6, 7 and 8. No digit can be used more than once in any one number. Find how many different 4-digit numbers can be formed if

(i) there are no restrictions, **840**
(ii) the number is even. **480**

3. Calculate the number of different 6-digit numbers which can be formed using the digits 0, 1, 2, 3, 4, 5 without repetition and assuming that a number cannot begin with 0.

4. The digits of the number 1 244 687 can be rearranged to give many different 7-digit numbers. How many of these 7-digit numbers are even? [4] **1800**

5. How many different numbers between 20 000 and 30 000 can be formed using 5 different digits from the digits 1, 2, 4, 6, 7, 8? [2] **120**

6.

(a)

i. Find how many different four-digit numbers can be made using only the digits 1, 3, 5 and 6 with no digit being repeated. [1] **24**
ii. Find how many different odd numbers greater than 500 can be made using some or all of the digits 1, 3, 5 and 6 with no digit being repeated. [4] **28**
iii. Six cards numbered 1, 2, 3, 4, 5, 6 are arranged randomly in a line. Find the probability that the cards numbered 4 and 5 are not next to each other. [3] **2/3**

7. Find how many different numbers can be made from some or all of the digits of the number 1 345 789 if

(i) all seven digits are used, the odd digits are all together and no digits are repeated, [2] **720**
(ii) the numbers made are even numbers between 3000 and 5000, and no digits are repeated, [3] **60**
(iii) the numbers made are multiples of 5 which are less than 1000, and digits can be repeated. [3] **57**

PERMUTATIONS IN A LINE

1.

a. A team of 3 boys and 3 girls is to be chosen from a group of 12 boys and 9 girls to enter a competition. Tom and Henry are two of the boys in the group. Find the number of ways in which the team can be chosen if Tom and Henry are either both in the team or both not in the team. [3] **10920**

b. The back row of a cinema has 12 seats, all of which are empty. A group of 8 people, including Mary and Frances, sit in this row. Find the number of different ways they can sit in these 12 seats if

(i) there are no restrictions, [1] **19958400**
(ii) Mary and Frances do not sit in seats which are next to each other, [3] **16632000**
(iii) all 8 people sit together with no empty seats between them. [3] **201600**

2. Four families go to a theme park together. Mr and Mrs Lin take their 2 children. Mr O'Connor takes his 2 children. Mr and Mrs Ahmed take their 3 children. Mrs Burton takes her son. The 14 people all have to go through a turnstile one at a time to enter the theme park.

(i) In how many different orders can the 14 people go through the turnstile if each family stays together? [3] **829440**

(ii) In how many different orders can the 8 children and 6 adults go through the turnstile if no two adults go consecutively? [3] $\mathbf{2.44 \times 10^9}$

Once inside the theme park, the children go on the roller-coaster. Each roller-coaster car holds 3 people.

(iii) In how many different ways can the 8 children be divided into two groups of 3 and one group of 2 to go on the roller-coaster?

PERMUTATIONS AND COMBINATIONS– VARIANT 62

1 (i) The height of sunflowers follows a normal distribution with mean 112 cm and standard deviati 17.2 cm. Find the probability that the height of a randomly chosen sunflower is greater th 120 cm. [

(ii) When a new fertiliser is used, the height of sunflowers follows a normal distribution with me 115 cm. Given that 80% of the heights are now greater than 103 cm, find the standard deviatio [

Answers: (i) 0.321; (ii) 14.3 . **J03/Q3**

---

2 A committee of 5 people is to be chosen from 6 men and 4 women. In how many ways can this done

(i) if there must be 3 men and 2 women on the committee, [

(ii) if there must be more men than women on the committee, [

(iii) if there must be 3 men and 2 women, and one particular woman refuses to be on the committ with one particular man? [

Answers: (i) 120; (ii) 186; (iii) 90. **J03/Q5**

---

3 (a) A football team consists of 3 players who play in a defence position, 3 players who play in midfield position and 5 players who play in a forward position. Three players are chosen t collect a gold medal for the team. Find in how many ways this can be done

(i) if the captain, who is a midfield player, must be included, together with one defence and on forward player, [2

(ii) if exactly one forward player must be included, together with any two others. [2

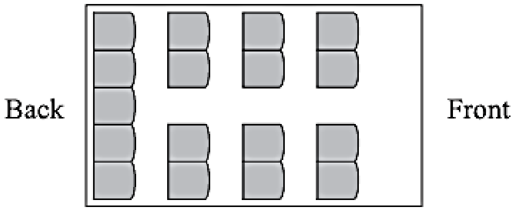
(b) Find how many different arrangements there are of the nine letters in the words GOLD MEDA

(i) if there are no restrictions on the order of the letters, [2

(ii) if the two letters D come first and the two letters L come last. [2

Answers: (i) 15; (ii) 75; (iii) 90 720; (iv) 120. **J05/Q7**

4



The diagram shows the seating plan for passengers in a minibus, which has 17 seats arranged in 4 row The back row has 5 seats and the other 3 rows have 2 seats on each side. 11 passengers get on t minibus.

(i) How many possible seating arrangements are there for the 11 passengers? [

(ii) How many possible seating arrangements are there if 5 particular people sit in the back row? [

Of the 11 passengers, 5 are unmarried and the other 6 consist of 3 married couples.

(iii) In how many ways can 5 of the 11 passengers on the bus be chosen if there must be 2 marrie couples and 1 other person, who may or may not be married? [

Answers: (i) 4.94 × 10^{11}; (ii) 79 833 600; (iii) 21. **J06/Q4**

5 **(i)** Find the number of ways in which all twelve letters of the word REFRIGERATOR can arranged

(a) if there are no restrictions, [

(b) if the Rs must all be together. [

(ii) How many different selections of four letters from the twelve letters of the word REFRIGERATO contain no Rs and two Es? [

Answers: (i)(a) 9 979 200, (b) 181 440; (ii) 15. **J07/Q5**

6 Issam has 11 different CDs, of which 6 are pop music, 3 are jazz and 2 are classical.

(i) How many different arrangements of all 11 CDs on a shelf are there if the jazz CDs are all ne to each other? [

(ii) Issam makes a selection of 2 pop music CDs, 2 jazz CDs and 1 classical CD. How many differe possible selections can be made? [

Answers: (i) 2 177 280; (ii) 90. **J08/Q3**

7 A choir consists of 13 sopranos, 12 altos, 6 tenors and 7 basses. A group consisting of 10 sopran 9 altos, 4 tenors and 4 basses is to be chosen from the choir.

(i) In how many different ways can the group be chosen? [

(ii) In how many ways can the 10 chosen sopranos be arranged in a line if the 6 tallest stand next each other? [

(iii) The 4 tenors and 4 basses in the group stand in a single line with all the tenors next to each oth and all the basses next to each other. How many possible arrangements are there if three of t tenors refuse to stand next to any of the basses? [

Answers: (i) 33 033 000; (ii) 86400; (iii) 288. **J09/Q4**

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8 Nine cards, each of a different colour, are to be arranged in a line.

(i) How many different arrangements of the 9 cards are possible? [

The 9 cards include a pink card and a green card.

(ii) How many different arrangements do not have the pink card next to the green card? [

Consider all possible choices of 3 cards from the 9 cards with the 3 cards being arranged in a line.

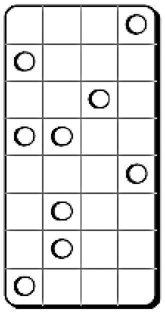
(iii) How many different arrangements in total of 3 cards are possible? [

(iv) How many of the arrangements of 3 cards in part **(iii)** contain the pink card? [

(v) How many of the arrangements of 3 cards in part **(iii)** do not have the pink card next to the gre card? [

Answers: (i) 362 880; (ii) 282 240; (iii) 504; (iv) 168; (v) 476. **J10/62/Q7**

9 In a certain hotel, the lock on the door to each room can be opened by inserting a key card. The ke card can be inserted only one way round. The card has a pattern of holes punched in it. The card h 4 columns, and each column can have either 1 hole, 2 holes, 3 holes or 4 holes punched in it. Eac column has 8 different positions for the holes. The diagram illustrates one particular key card wi 3 holes punched in the first column, 3 in the second, 1 in the third and 2 in the fourth.



(i) Show that the number of different ways in which a column could have exactly 2 holes is 2 [

(ii) Find how many different patterns of holes can be punched in a column. [

(iii) How many different possible key cards are there? [

Answers: (ii) 162; (iii) 688 747 536 or 689 000 000. **N02/Q4**

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10 **(a)** A collection of 18 books contains one Harry Potter book. Linda is going to choose 6 of thes books to take on holiday.

(i) In how many ways can she choose 6 books? [

(ii) How many of these choices will include the Harry Potter book? [

(b) In how many ways can 5 boys and 3 girls stand in a straight line

(i) if there are no restrictions, [

(ii) if the boys stand next to each other? [

Answers: (a)(i) 18 564, (ii) 6188; (b)(i) 40 320, (ii) 2880. **N03/Q6**

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11 The word ARGENTINA includes the four consonants R, G, N, T and the three vowels A, E, I.

(i) Find the number of different arrangements using all nine letters. [

(ii) How many of these arrangements have a consonant at the beginning, then a vowel, then anoth consonant, and so on alternately? [

Answers: (i) 90 720; (ii) 720. **N04/Q1**

12 A staff car park at a school has 13 parking spaces in a row. There are 9 cars to be parked.

(i) How many different arrangements are there for parking the 9 cars and leaving 4 empty spaces [

(ii) How many different arrangements are there if the 4 empty spaces are next to each other? [

(iii) If the parking is random, find the probability that there will **not** be 4 empty spaces next to eac other. [

Answers: (i) 259 459 200; (ii) 3 628 800; (iii) 0.986. **N05/Q3**

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13 Six men and three women are standing in a supermarket queue.

(i) How many possible arrangements are there if there are no restrictions on order? [

(ii) How many possible arrangements are there if no two of the women are standing next to eac other? [

(iii) Three of the people in the queue are chosen to take part in a customer survey. How many differe choices are possible if at least one woman must be included? [

Answers: (i) 362 880; (ii) 151 200; (iii) 64. **N06/Q6**

---

14 The six digits 4, 5, 6, 7, 7, 7 can be arranged to give many different 6-digit numbers.

(i) How many different 6-digit numbers can be made? [

(ii) How many of these 6-digit numbers start with an odd digit and end with an odd digit? [

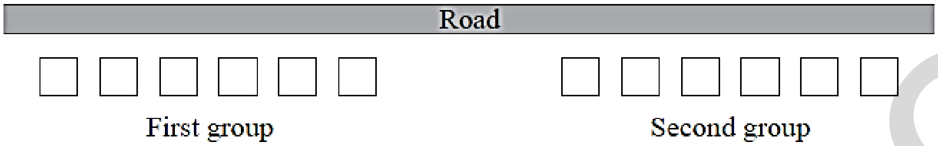
N07/Q3

Answers: (i) 120; (ii) 48.

15 A builder is planning to build 12 houses along one side of a road. He will build 2 houses in style 2 houses in style B, 3 houses in style C, 4 houses in style D and 1 house in style E.

(i) Find the number of possible arrangements of these 12 houses. [

(ii)



The 12 houses will be in two groups of 6 (see diagram). Find the number of possible arrangemen if all the houses in styles A and D are in the first group and all the houses in styles B, C and are in the second group. [

(iii) Four of the 12 houses will be selected for a survey. Exactly one house must be in style B an exactly one house in style C. Find the number of ways in which these four houses can be selecte [

Answers: (i) 831 600; (ii) 900; (iii) 126. **N08/Q4**

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16 (a) (i) Find how many different four-digit numbers can be made using only the digits 1, 3, 5 and with no digit being repeated. [

(ii) Find how many different odd numbers greater than 500 can be made using some or all the digits 1, 3, 5 and 6 with no digit being repeated. [

(b) Six cards numbered 1, 2, 3, 4, 5, 6 are arranged randomly in a line. Find the probability that t cards numbered 4 and 5 are **not** next to each other. [

\- *Answers*: (a)(i) 24, (ii) 28; (b) $\frac{2}{3}$. **N09/62/Q4**

---

17 A committee of 6 people, which must contain at least 4 men and at least 1 woman, is to be chos from 10 men and 9 women.

(i) Find the number of possible committees that can be chosen. [

(ii) Find the probability that one particular man, Albert, and one particular woman, Tracey, are bo on the committee. [

(iii) Find the number of possible committees that include either Albert or Tracey but not both. [

(iv) The committee that is chosen consists of 4 men and 2 women. They queue up randomly in a li for refreshments. Find the probability that the women are not next to each other in the queue. [

Answers: (i) 9828; (ii) $\frac{798}{9828}$ (0.0812); (iii) 4494; (iv) $\frac{2}{3}$ (0.667). **N10/62/Q7**

18 **(i)** Find the number of different ways that the 9 letters of the word HAPPINESS can be arranged
a line. [

(ii) The 9 letters of the word HAPPINESS are arranged in random order in a line. Find the probabili
that the 3 vowels (A, E, I) are not all next to each other. [

(iii) Find the number of different selections of 4 letters from the 9 letters of the word HAPPINES
which contain no Ps and either one or two Ss. [

Answers: (i) 90 720; (ii) 0.917; (iii) 20. **J11/62/Q4**

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19 An English examination consists of 8 questions in Part *A* and 3 questions in Part *B*. Candidates mu
choose 6 questions. The order in which questions are chosen does not matter. Find the number
ways in which the 6 questions can be chosen in each of the following cases.

(i) There are no restrictions on which questions can be chosen. [

(ii) Candidates must choose at least 4 questions from Part *A*. [

(iii) Candidates must either choose both question 1 and question 2 in Part *A*, or choose neither
these questions. [

Answers: (i) 462 (ii) 406 (iii) 210 **J12/62/Q5**

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20 A town council plans to plant 12 trees along the centre of a main road. The council buys the tre
from a garden centre which has 4 different hibiscus trees, 9 different jacaranda trees and 2 differe
oleander trees for sale.

(i) How many different selections of 12 trees can be made if there must be at least 2 of each type
tree? [

The council buys 4 hibiscus trees, 6 jacaranda trees and 2 oleander trees.

(ii) How many different arrangements of these 12 trees can be made if the hibiscus trees have to
next to each other, the jacaranda trees have to be next to each other and the oleander trees hav
to be next to each other? [

(iii) How many different arrangements of these 12 trees can be made if no hibiscus tree is next
another hibiscus tree? [

Answers: 282, 207360, 121,927,680 **J13/62/Q6**

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21 A school club has members from 3 different year-groups: Year 1, Year 2 and Year 3. There are
members from Year 1, 2 members from Year 2 and 2 members from Year 3. Five members of t
club are selected. Find the number of possible selections that include at least one member from ea
year-group. [

Answer: 231 **J14/62/Q2**

22 Find how many different numbers can be made from some or all of the digits of the number 1 345 7 if

(i) all seven digits are used, the odd digits are all together and no digits are repeated, [

(ii) the numbers made are even numbers between 3000 and 5000, and no digits are repeated, [

(iii) the numbers made are multiples of 5 which are less than 1000, and digits can be repeated. [

Answers: 720, 60, 57 **J14/62/Q5**

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23 **(a)** Find the number of different ways the 7 letters of the word BANANAS can be arranged

(i) if the first letter is N and the last letter is B, [

(ii) if all the letters A are next to each other. [

(b) Find the number of ways of selecting a group of 9 people from 14 if two particular people cann both be in the group together. [

Answers:**(a)(i)** 20, **(ii)** 60, **(b)** 1210 **J15/62/Q6**

---

24 Twelve coins are tossed and placed in a line. Each coin can show either a head or a tail.

(i) Find the number of different arrangements of heads and tails which can be obtained. [

(ii) Find the number of different arrangements which contain 7 heads and 5 tails. [

Answers: **(i)** 4,096; **(ii)** 792 **N11/62/Q2**

---

25 **(a)** Geoff wishes to plant 25 flowers in a flower-bed. He can choose from 15 different geraniu 10 different roses and 8 different lilies. He wants to have at least 11 geraniums and also to ha the same number of roses and lilies. Find the number of different selections of flowers he c make. [

(b) Find the number of different ways in which the 9 letters of the word GREENGAGE can arranged if exactly two of the Gs are next to each other. [

Answers: **(i)** 1,941,912; **(ii)** 5040 **N11/62/Q3**

26 **(a)** A team of 3 boys and 3 girls is to be chosen from a group of 12 boys and 9 girls to enter competition. Tom and Henry are two of the boys in the group. Find the number of ways in whic the team can be chosen if Tom and Henry are either both in the team or both not in the team. [

(b) The back row of a cinema has 12 seats, all of which are empty. A group of 8 people, includin Mary and Frances, sit in this row. Find the number of different ways they can sit in these 1 seats if

- **(i)** there are no restrictions, [
- **(ii)** Mary and Frances do not sit in seats which are next to each other, [
- **(iii)** all 8 people sit together with no empty seats between them. [

Answers: (a) 10920; (b)(i) 19958400; (ii) 16632000; (iii) 201600. **N12/62/Q5**

27 The 11 letters of the word REMEMBRANCE are arranged in a line.

- **(i)** Find the number of different arrangements if there are no restrictions. [
- **(ii)** Find the number of different arrangements which start and finish with the letter M. [
- **(iii)** Find the number of different arrangements which do not have all 4 vowels (E, E, A, E) next t each other. [

4 letters from the letters of the word REMEMBRANCE are chosen.

- **(iv)** Find the number of different selections which contain no Ms and no Rs and at least 2 Es. [

Answers: 1663200, 30240, 1622880, 10 **N13/62/Q6**

28 The 50 members of a club include both the club president and the club treasurer. All 50 membe want to go on a coach tour, but the coach only has room for 45 people. In how many ways can members be chosen if both the club president and the club treasurer must be included? [

Answer: $^{8}C_{43} = 1712304$ **N14/62/Q1**

29 Find the number of different ways that 6 boys and 4 girls can stand in a line if

- **(i)** all 6 boys stand next to each other, [
- **(ii)** no girl stands next to another girl. [

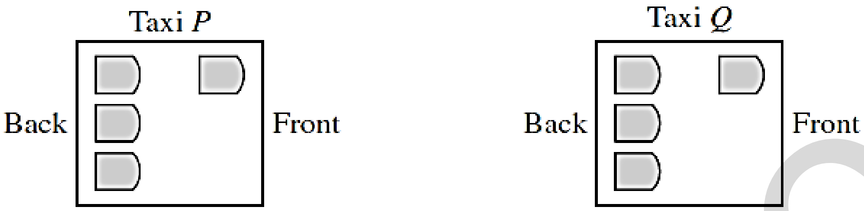
Answers: 86400, 604800 **N14/62/Q2**

30 A committee of 6 people is to be chosen at random from 7 men and 9 women. Find the probabilit that there are no men on the committee. [

Answer: 3/286 (0.0105) **N15/62/Q2**

31 A group of 8 friends travels to the airport in two taxis, P and Q. Each taxi can take 4 passengers.

(i) The 8 friends divide themselves into two groups of 4, one group for taxi P and one group for ta Q, with Jon and Sarah travelling in the same taxi. Find the number of different ways in whic this can be done. [



Each taxi can take 1 passenger in the front and 3 passengers in the back (see diagram). Mark sits i the front of taxi P and Jon and Sarah sit in the back of taxi P next to each other.

(ii) Find the number of different seating arrangements that are now possible for the 8 friends. [

Answers: **(i)** 30, **(ii)** 480.

N15/62/Q4

PERMUTATIONS AND COMBINATIONS VARIANT 61 AND 63

1 (a) Find how many different numbers can be made by arranging all nine digits of the number 223 677 888 if

(i) there are no restrictions, [2]

(ii) the number made is an even number. [4]

(b) Sandra wishes to buy some applications (apps) for her smartphone but she only has enough money for 5 apps in total. There are 3 train apps, 6 social network apps and 14 games apps available. Sandra wants to have at least 1 of each type of app. Find the number of different possible selections of 5 apps that Sandra can choose. [5]

Answer: i) 15120 ii)10080 b)13839 **61/J15/7**

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2 Rachel has 3 types of ornament. She has 6 different wooden animals, 4 different sea-shells and 3 different pottery ducks.

(i) She lets her daughter Cherry choose 5 ornaments to play with. Cherry chooses at least 1 of each type of ornament. How many different selections can Cherry make? [5]

Rachel displays 10 of the 13 ornaments in a row on her window-sill. Find the number of different arrangements that are possible if

(ii) she has a duck at each end of the row and no ducks anywhere else, [3]

(iii) she has a duck at each end of the row and wooden animals and sea-shells are placed alternately in the positions in between. [3]

Answer: i) 894 ii)10886400 iii)103680 **63/J15/7**

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3 (a) Find the number of ways in which all nine letters of the word TENNESSEE can be arranged

(i) if all the letters E are together, [3]

(ii) if the T is at one end and there is an S at the other end. [3]

(b) Four letters are selected from the nine letters of the word VENEZUELA. Find the number of possible selections which contain exactly one E. [3]

Answer:i) 180 ii) 210 iii) 20 **63/N15/5**

4 The faces of a biased die are numbered 1, 2, 3, 4, 5 and 6. The probabilities of throwing odd numbers are all the same. The probabilities of throwing even numbers are all the same. The probability of throwing an odd number is twice the probability of throwing an even number.

(i) Find the probability of throwing a 3. [3]

(ii) The die is thrown three times. Find the probability of throwing two 5s and one 4. [3]

(iii) The die is thrown 100 times. Use an approximation to find the probability that an even number is thrown at most 37 times. [5]

Answer: i) 0.222 ii) 0.0165 iii) 0.812 **61/N15/7**

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5 (a) Find the number of different ways that the 13 letters of the word ACCOMMODATION can be arranged in a line if all the vowels (A, I, O) are next to each other. [3]

(b) There are 7 Chinese, 6 European and 4 American students at an international conference. Four of the students are to be chosen to take part in a television broadcast. Find the number of different ways the students can be chosen if at least one Chinese and at least one European student are included. [5]

Answer: i) 604800 ii) 1841 **61/N15/5**

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6 (a) Seven fair dice each with faces marked 1, 2, 3, 4, 5, 6 are thrown and placed in a line. Find the number of possible arrangements where the sum of the numbers at each end of the line add up to 4. [3]

(b) Find the number of ways in which 9 different computer games can be shared out between Wainah, Jingyi and Hebe so that each person receives an odd number of computer games. [6]

Answer: a) 23328 b) 4920 **63/N14/6**

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7 The 50 members of a club include both the club president and the club treasurer. All 50 members want to go on a coach tour, but the coach only has room for 45 people. In how many ways can 45 members be chosen if both the club president and the club treasurer must be included? [3]

Answer: 1712304 **62/N14/1**

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8 Find the number of different ways that 6 boys and 4 girls can stand in a line if

(i) all 6 boys stand next to each other, [3]

(ii) no girl stands next to another girl. [3]

Answer: i) 86400 ii) 604800 **62/N14/2**

9 A committee of 6 people is to be chosen from 5 men and 8 women. In how many ways can this be done

(i) if there are more women than men on the committee, [4]

(ii) if the committee consists of 3 men and 3 women but two particular men refuse to be on the committee together? [3]

One particular committee consists of 5 women and 1 man.

(iii) In how many different ways can the committee members be arranged in a line if the man is not at either end? [3]

Answer: i) 1008 ii) 392 iii) 480 **61/N14/7**

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10 Nine cards are numbered 1, 2, 2, 3, 3, 4, 6, 6, 6.

(i) All nine cards are placed in a line, making a 9-digit number. Find how many different 9-digit numbers can be made in this way

(a) if the even digits are all together, [4]

(b) if the first and last digits are both odd. [3]

(ii) Three of the nine cards are chosen and placed in a line, making a 3-digit number. Find how many different numbers can be made in this way

(a) if there are no repeated digits, [2]

(b) if the number is between 200 and 300. [2]

Answer: i) 720 ii) 1260 iii) 60 iv) 22 **63/J14/7**

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11 Find the number of different ways in which all 8 letters of the word TANZANIA can be arranged so that

(i) all the letters A are together, [2]

(ii) the first letter is a consonant (T, N, Z), the second letter is a vowel (A, I), the third letter is a consonant, the fourth letter is a vowel, and so on alternately. [3]

4 of the 8 letters of the word TANZANIA are selected. How many possible selections contain

(iii) exactly 1 N and 1 A, [2]

(iv) exactly 1 N? [3]

Answer: i) 360, ii) 48 iii) 3 iv) 8 **61/J14/6**

12 (i) Find the number of different ways that the 9 letters of the word AGGREGATE can be arranged in a line if the first letter is R. [2]

(ii) Find the number of different ways that the 9 letters of the word AGGREGATE can be arranged in a line if the 3 letters G are together, both letters A are together and both letters E are together. [2]

(iii) The letters G, R and T are consonants and the letters A and E are vowels. Find the number of different ways that the 9 letters of the word AGGREGATE can be arranged in a line if consonants and vowels occur alternately. [3]

(iv) Find the number of different selections of 4 letters of the word AGGREGATE which contain exactly 2 Gs or exactly 3 Gs. [3]

Answer: i) 1680, ii) 120 iii) 120 iv) 12

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13 A shop has 7 different mountain bicycles, 5 different racing bicycles and 8 different ordinary bicycles on display. A cycling club selects 6 of these 20 bicycles to buy.

(i) How many different selections can be made if there must be no more than 3 mountain bicycles and no more than 2 of each of the other types of bicycle? [4]

The cycling club buys 3 mountain bicycles, 1 racing bicycle and 2 ordinary bicycles and parks them in a cycle rack, which has a row of 10 empty spaces.

(ii) How many different arrangements are there in the cycle rack if the mountain bicycles are all together with no spaces between them, the ordinary bicycles are both together with no spaces between them and the spaces are all together? [3]

(iii) How many different arrangements are there in the cycle rack if the ordinary bicycles are at each end of the bicycles and there are no spaces between any of the bicycles? [3]

Answer: i) 13580 ii) 288 iii) 240 **61/N13/6**

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14 There are 10 spaniels, 14 retrievers and 6 poodles at a dog show. 7 dogs are selected to go through to the final.

(i) How many selections of 7 different dogs can be made if there must be at least 1 spaniel, at least 2 retrievers and at least 3 poodles? [4]

2 spaniels, 2 retrievers and 3 poodles go through to the final. They are placed in a line.

(ii) How many different arrangements of these 7 dogs are there if the spaniels stand together and the retrievers stand together? [3]

(iii) How many different arrangements of these 7 dogs are there if no poodle is next to another poodle? [3]

Answer: i) 168350 ii) 480 iii) 1440 **63/J13/7**

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15 Four families go to a theme park together. Mr and Mrs Lin take their 2 children. Mr O'Connor takes his 2 children. Mr and Mrs Ahmed take their 3 children. Mrs Burton takes her son. The 14 people all have to go through a turnstile one at a time to enter the theme park.

(i) In how many different orders can the 14 people go through the turnstile if each family stays together? [3]

(ii) In how many different orders can the 8 children and 6 adults go through the turnstile if no two adults go consecutively? [3]

Once inside the theme park, the children go on the roller-coaster. Each roller-coaster car holds 3 people.

(iii) In how many different ways can the 8 children be divided into two groups of 3 and one group of 2 to go on the roller-coaster? [3]

Answer: i) 829440 ii) 2438553600 iii) 560 **61/J13/6**

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16 (a) A chess team of 2 girls and 2 boys is to be chosen from the 7 girls and 6 boys in the chess club. Find the number of ways this can be done if 2 of the girls are twins and are either both in the team or both not in the team. [3]

(b) (i) The digits of the number 1 244 687 can be rearranged to give many different 7-digit numbers. How many of these 7-digit numbers are even? [4]

(ii) How many different numbers between 20 000 and 30 000 can be formed using 5 different digits from the digits 1, 2, 4, 6, 7, 8? [2]

(c) Helen has some black tiles, some white tiles and some grey tiles. She places a single row of 8 tiles above her washbasin. Each tile she places is equally likely to be black, white or grey. Find the probability that there are no tiles of the same colour next to each other. [3]

Answer: i) 165 ii) 1800 iii) 120 iv) 0.0585 **63/N12/6**

17 (a) In a sweet shop 5 identical packets of toffees, 4 identical packets of fruit gums and 9 identical packets of chocolates are arranged in a line on a shelf. Find the number of different arrangements of the packets that are possible if the packets of chocolates are kept together. [2]

(b) Jessica buys 8 different packets of biscuits. She then chooses 4 of these packets.

(i) How many different choices are possible if the order in which Jessica chooses the 4 packets is taken into account? [2]

The 8 packets include 1 packet of chocolate biscuits and 1 packet of custard creams.

(ii) How many different choices are possible if the order in which Jessica chooses the 4 packets is taken into account and the packet of chocolate biscuits and the packet of custard creams are both chosen? [3]

(c) 9 different fruit pies are to be divided between 3 people so that each person gets an odd number of pies. Find the number of ways this can be done. [5]

Answer: i) 1260 ii) 1680 iii) 360 iv) 4920 **61/N12/7**

---

18 (i) In how many ways can all 9 letters of the word TELEPHONE be arranged in a line if the letters P and L must be at the ends? [2]

How many different selections of 4 letters can be made from the 9 letters of the word TELEPHONE if

(ii) there are no Es, [1]

(iii) there is exactly 1 E, [2]

(iv) there are no restrictions? [4]

Answer: i) 1680 ii) 15 iii) 20 iv) 56 **63/J12/3**

---

19 (a) Seven friends together with their respective partners all meet up for a meal. To commemorate the occasion they arrange for a photograph to be taken of all 14 of them standing in a line.

(i) How many different arrangements are there if each friend is standing next to his or her partner? [3]

(ii) How many different arrangements are there if the 7 friends all stand together and the 7 partners all stand together? [2]

(b) A group of 9 people consists of 2 boys, 3 girls and 4 adults. In how many ways can a team of 4 be chosen if

(i) both boys are in the team, [1]

(ii) the adults are either all in the team or all not in the team, [2]

(iii) at least 2 girls are in the team? [2]

Answer: i) 645120 ii) 50803200 iii) 21 iv) 6 v) 51 **61/J12/7**

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20 Mary saves her digital images on her computer in three separate folders named 'Family', 'Holiday' and 'Friends'. Her family folder contains 3 images, her holiday folder contains 4 images and her friends folder contains 8 images. All the images are different.

(i) Find in how many ways she can arrange these 15 images in a row across her computer screen if she keeps the images from each folder together. [3]

(ii) Find the number of different ways in which Mary can choose 6 of these images if there are 2 from each folder. [2]

(iii) Find the number of different ways in which Mary can choose 6 of these images if there are at least 3 images from the friends folder and at least 1 image from each of the other two folders. [4]

Answer: i) 34836480 ii) 504 iii) 2520 **63/N11/4**

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21 (a) Find the number of different ways in which the 12 letters of the word STRAWBERRIES can be arranged

(i) if there are no restrictions, [2]

(ii) if the 4 vowels A, E, E, I must all be together. [3]

(b) (i) 4 astronauts are chosen from a certain number of candidates. If order of choosing is not taken into account, the number of ways the astronauts can be chosen is 3876. How many ways are there if order of choosing is taken into account? [2]

(ii) 4 astronauts are chosen to go on a mission. Each of these astronauts can take 3 personal possessions with him. How many different ways can these 12 possessions be arranged in a row if each astronaut's possessions are kept together? [2]

Answer: i) 19958400 ii) 362880 iii) 93024 iv) 31104 **61/J11/6**

22 Fahad has 4 different coloured pairs of shoes (white, red, blue and black), 3 different coloured pairs of jeans (blue, black and brown) and 7 different coloured tee shirts (red, orange, yellow, blue, green, white and purple).

(i) Fahad chooses an outfit consisting of one pair of shoes, one pair of jeans and one tee shirt. How many different outfits can he choose? [1]

(ii) How many different ways can Fahad arrange his 3 jeans and 7 tee shirts in a row if the two blue items are not next to each other? [2]

Fahad also has 9 different books about sport. When he goes on holiday he chooses at least one of these books to take with him.

(iii) How many different selections are there if he can take any number of books ranging from just one of them to all of them? [3]

Answer: i) 84 ii) 2903040 iii) 511 **63/J11/2**

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23 A cricket team of 11 players is to be chosen from 21 players consisting of 10 batsmen, 9 bowlers and 2 wicketkeepers. The team must include at least 5 batsmen, at least 4 bowlers and at least 1 wicketkeeper.

(i) Find the number of different ways in which the team can be chosen. [4]

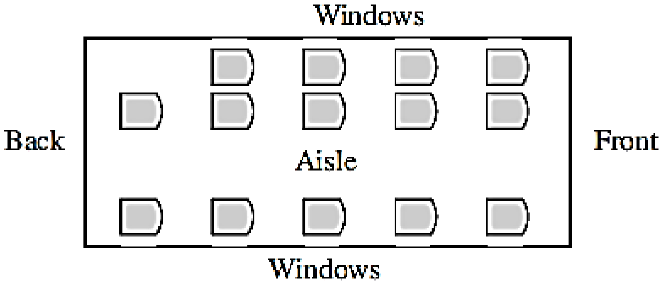
Each player in the team is given a present. The presents consist of 5 identical pens, 4 identical diaries and 2 identical notebooks.

(ii) Find the number of different arrangements of the presents if they are all displayed in a row. [1]

(iii) 10 of these 11 presents are chosen and arranged in a row. Find the number of different arrangements that are possible. [3]

Answer: i) 148176 ii) 6930 iii) 6930 **61/J11/4**

24



A small aeroplane has 14 seats for passengers. The seats are arranged in 4 rows of 3 seats and a back row of 2 seats (see diagram). 12 passengers board the aeroplane.

(i) How many possible seating arrangements are there for the 12 passengers? Give your answer correct to 3 significant figures. [2]

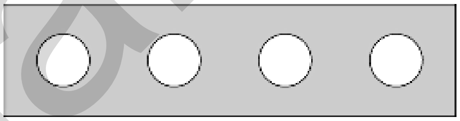
These 12 passengers consist of 2 married couples (Mr and Mrs Lin and Mr and Mrs Brown), 5 students and 3 business people.

(ii) The 3 business people sit in the front row. The 5 students each sit at a window seat. Mr and Mrs Lin sit in the same row on the same side of the aisle. Mr and Mrs Brown sit in another row on the same side of the aisle. How many possible seating arrangements are there? [4]

(iii) If, instead, the 12 passengers are seated randomly, find the probability that Mrs Lin sits directly behind a student and Mrs Brown sits in the front row. [4]

Answer: i) 4.36 x 10^10 ii) 17280 iii) 0.0687 **63/N10/6**

25



Pegs are to be placed in the four holes shown, one in each hole. The pegs come in different colours and pegs of the same colour are identical. Calculate how many different arrangements of coloured pegs in the four holes can be made using

(i) 6 pegs, all of different colours, [1]

(ii) 4 pegs consisting of 2 blue pegs, 1 orange peg and 1 yellow peg. [1]

Beryl has 12 pegs consisting of 2 red, 2 blue, 2 green, 2 orange, 2 yellow and 2 black pegs. Calculate how many different arrangements of coloured pegs in the 4 holes Beryl can make using

(iii) 4 different colours, [1]

(iv) 3 different colours, [3]

(v) any of her 12 pegs. [3]

Answer: i) 360 ii) 12 iii) 360 iv) 720 v) 1170 **61/N10/6**

---

26 Three identical cans of cola, 2 identical cans of green tea and 2 identical cans of orange juice are arranged in a row. Calculate the number of arrangements if

(i) the first and last cans in the row are the same type of drink, [3]

(ii) the 3 cans of cola are all next to each other and the 2 cans of green tea are not next to each other. [5]

Answer: i) 50 ii) 18 **63/N10/4**

---

27 **(i)** Find the number of different ways that a set of 10 different mugs can be shared between Lucy and Monica if each receives an odd number of mugs. [3]

(ii) Another set consists of 6 plastic mugs each of a different design and 3 china mugs each of a different design. Find in how many ways these 9 mugs can be arranged in a row if the china mugs are all separated from each other. [3]

(iii) Another set consists of 3 identical red mugs, 4 identical blue mugs and 7 identical yellow mugs. These 14 mugs are placed in a row. Find how many different arrangements of the colours are possible if the red mugs are kept together. [3]

Answer: i) 512 ii) 151200 iii) 3960 **61/J10/6**

---

28 **(a)** **(i)** Find how many numbers there are between 100 and 999 in which all three digits are different. [3]

(ii) Find how many of the numbers in part **(i)** are odd numbers greater than 700. [4]

(b) A bunch of flowers consists of a mixture of roses, tulips and daffodils. Tom orders a bunch of 7 flowers from a shop to give to a friend. There must be at least 2 of each type of flower. The shop has 6 roses, 5 tulips and 4 daffodils, all different from each other. Find the number of different bunches of flowers that are possible. [4]

Answers: (i) 648 (ii) 104 (iii) 2700 **J16/61/Q6**

29 Find the number of ways all 9 letters of the word EVERGREEN can be arranged if

(i) there are no restrictions, [1]

(ii) the first letter is R and the last letter is G, [2]

(iii) the Es are all together. [2]

Three letters from the 9 letters of the word EVERGREEN are selected.

(iv) Find the number of selections which contain no Es and exactly 1 R. [1]

(v) Find the number of selections which contain no Es. [3]

Answers: (i) 7 560 (ii) 210 (iii) 360 (iv) 3 (v) 7 **J16/63/Q6**

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30 **(a)** Eight children of different ages stand in a random order in a line. Find the number of different ways this can be done if none of the three youngest children stand next to each other. [3]

(b) David chooses 5 chocolates from 6 different dark chocolates, 4 different white chocolates and 1 milk chocolate. He must choose at least one of each type. Find the number of different selections he can make. [4]

(c) A password for Chelsea's computer consists of 4 characters in a particular order. The characters are chosen from the following.

- The 26 capital letters A to Z
- The 9 digits 1 to 9
- The 5 symbols # ~ * ? !

The password must include at least one capital letter, at least one digit and at least one symbol. No character can be repeated. Find the number of different passwords that Chelsea can make. [4]

Answer: (a) 14400 (b) 194 (c) 519480 **J17/61/Q7**

31 **(a)** Find how many numbers between 3000 and 5000 can be formed from the digits 1, 2, 3, 4 and 5,

(i) if digits are not repeated, [2]

(ii) if digits can be repeated and the number formed is odd. [3]

(b) A box of 20 biscuits contains 4 different chocolate biscuits, 2 different oatmeal biscuits and 14 different ginger biscuits. 6 biscuits are selected from the box at random.

(i) Find the number of different selections that include the 2 oatmeal biscuits. [2]

(ii) Find the probability that fewer than 3 chocolate biscuits are selected. [4]

Answers: (a)(i) 48 (ii) 150 (b)(i) 3060 (ii) 0.939 **J17/63/Q6**

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32 **(a)** Find the number of different ways of arranging all nine letters of the word PINEAPPLE if no vowel (A, E, I) is next to another vowel. [4]

(b) A certain country has a cricket squad of 16 people, consisting of 7 batsmen, 5 bowlers, 2 all-rounders and 2 wicket-keepers. The manager chooses a team of 11 players consisting of 5 batsmen, 4 bowlers, 1 all-rounder and 1 wicket-keeper.

(i) Find the number of different teams the manager can choose. [2]

(ii) Find the number of different teams the manager can choose if one particular batsman refuses to be in the team when one particular bowler is in the team. [3]

Answers: (a) 3600 (b)(i) 420 (ii) 180 **N16/61/Q5**

---

33 A committee of 5 people is to be chosen from 4 men and 6 women. William is one of the 4 men and Mary is one of the 6 women. Find the number of different committees that can be chosen if William and Mary refuse to be on the committee together. [3]

Answer: 196 **N16/63/Q1**

---

34 Numbers are formed using some or all of the digits 4, 5, 6, 7 with no digit being used more than once.

(i) Show that, using exactly 3 of the digits, there are 12 different odd numbers that can be formed. [3]

(ii) Find how many odd numbers altogether can be formed. [3]

Answers: (ii) 32 **N16/63/Q3**

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35 **(a)** A village hall has seats for 40 people, consisting of 8 rows with 5 seats in each row. Mary, Ahmad, Wayne, Elsie and John are the first to arrive in the village hall and no seats are taken before they arrive.

(i) How many possible arrangements are there of seating Mary, Ahmad, Wayne, Elsie and John assuming there are no restrictions? [2]

(ii) How many possible arrangements are there of seating Mary, Ahmad, Wayne, Elsie and John if Mary and Ahmad sit together in the front row and the other three sit together in one of the other rows? [4]

(b) In how many ways can a team of 4 people be chosen from 10 people if 2 of the people, Ross and Lionel, refuse to be in the team together? [4]

Answers: (a)(i) 78960960 (ii) 1008 (iii) 182 **N17/61/Q6**

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36 A car park has spaces for 18 cars, arranged in a line. On one day there are 5 cars, of different makes, parked in randomly chosen positions and 13 empty spaces.

(i) Find the number of possible arrangements of the 5 cars in the car park. [2]

(ii) Find the probability that the 5 cars are not all next to each other. [5]

On another day, 12 cars of different makes are parked in the car park. 5 of these cars are red, 4 are white and 3 are black. Elizabeth selects 3 of these cars.

(iii) Find the number of selections Elizabeth can make that include cars of at least 2 different colours. [5]

Answers: (i) 1 028 160, (ii) 0.998, (iii) 205 **N17/63/Q6**

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37 9 people are to be divided into a group of 4, a group of 3 and a group of 2. In how many different ways can this be done? [3]

Answer: 1260 **N18/61/Q1**

38 In an orchestra, there are 11 violinists, 5 cellists and 4 double bass players. A small group of 6 musicians is to be selected from these 20.

(i) How many different selections of 6 musicians can be made if there must be at least 4 violinists, at least 1 cellist and no more than 1 double bass player? [4]

The small group that is selected contains 4 violinists, 1 cellist and 1 double bass player. They sit in a line to perform a concert.

(ii) How many different arrangements are there of these 6 musicians if the violinists must sit together? [3]

Answers: (i) 12 210 (ii) 144 **N18/61/Q3**

---

39 A group consists of 5 men and 2 women. Find the number of different ways that the group can stand in a line if the women are not next to each other. [3]

Answer: 3600 **N18/63/Q1**

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40 Out of a class of 8 boys and 4 girls, a group of 7 people is chosen at random.

(i) Find the probability that the group of 7 includes one particular boy. [3]

(ii) Find the probability that the group of 7 includes at least 2 girls. [4]

Answer: $\frac{7}{12}$ (0.583) *Answer:* $\frac{28}{33}$ (0.848) **N18/63/Q4**

5 Probability & Statistics 1 (for Paper 5)

Questions set will be mainly numerical, and will test principles in probability and statistics without involving knowledge of algebraic methods beyond the content for Paper 1: Pure Mathematics 1.

Knowledge of the following probability notation is also assumed: $\mathrm{P}(A)$, $\mathrm{P}(A \cup B)$, $P(A \cap B)$, $\mathrm{P}(A|B)$ and the use of A' to denote the complement of A.

5.1 Representation of data

Candidates should be able to:	Notes and examples
• select a suitable way of presenting raw statistical data, and discuss advantages and/or disadvantages that particular representations may have	
• draw and interpret stem-and-leaf diagrams, box-and-whisker plots, histograms and cumulative frequency graphs	Including back-to-back stem-and-leaf diagrams.
• understand and use different measures of central tendency (mean, median, mode) and variation (range, interquartile range, standard deviation)	e.g. in comparing and contrasting sets of data.
• use a cumulative frequency graph	e.g. to estimate medians, quartiles, percentiles, the proportion of a distribution above (or below) a given value, or between two values.
• calculate and use the mean and standard deviation of a set of data (including grouped data) either from the data itself or from given totals Σx and Σx^2, or coded totals $\Sigma(x-a)$ and $\Sigma(x-a)^2$, and use such totals in solving problems which may involve up to two data sets.	

5.2 Permutations and combinations

Candidates should be able to:	Notes and examples
• understand the terms permutation and combination, and solve simple problems involving selections	
• solve problems about arrangements of objects in a line, including those involving – repetition (e.g. the number of ways of arranging the letters of the word 'NEEDLESS') – restriction (e.g. the number of ways several people can stand in a line if two particular people must, or must not, stand next to each other).	Questions may include cases such as people sitting in two (or more) rows. Questions about objects arranged in a circle will not be included.

5 Probability & Statistics 1

5.3 Probability

Candidates should be able to:	Notes and examples
• evaluate probabilities in simple cases by means of enumeration of equiprobable elementary events, or by calculation using permutations or combinations	e.g. the total score when two fair dice are thrown. e.g. drawing balls at random from a bag containing balls of different colours.
• use addition and multiplication of probabilities, as appropriate, in simple cases	Explicit use of the general formula $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ is not required.
• understand the meaning of exclusive and independent events, including determination of whether events A and B are independent by comparing the values of $\mathrm{P}(A \cap B)$ and $\mathrm{P}(A) \times \mathrm{P}(B)$	
• calculate and use conditional probabilities in simple cases.	e.g. situations that can be represented by a sample space of equiprobable elementary events, or a tree diagram. The use of $\mathrm{P}(A\|B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$ may be required in simple cases.

5.4 Discrete random variables

Candidates should be able to:	Notes and examples
• draw up a probability distribution table relating to a given situation involving a discrete random variable X, and calculate $\mathrm{E}(X)$ and $\mathrm{Var}(X)$	
• use formulae for probabilities for the binomial and geometric distributions, and recognise practical situations where these distributions are suitable models	Including the notations $\mathrm{B}(n, p)$ and $\mathrm{Geo}(p)$. $\mathrm{Geo}(p)$ denotes the distribution in which $p_r = p(1-p)^{r-1}$ for $r = 1, 2, 3, \dots$.
• use formulae for the expectation and variance of the binomial distribution and for the expectation of the geometric distribution.	Proofs of formulae are not required.

5 Probability & Statistics 1

5.5 The normal distribution

Candidates should be able to:	Notes and examples
• understand the use of a normal distribution to model a continuous random variable, and use normal distribution tables	Sketches of normal curves to illustrate distributions or probabilities may be required.
• solve problems concerning a variable X, where $X \sim N(\mu, \sigma^2)$, including – finding the value of $P(X > x_1)$, or a related probability, given the values of x_1, μ, σ. – finding a relationship between x_1, μ and σ given the value of $P(X > x_1)$ or a related probability	For calculations involving standardisation, full details of the working should be shown. e.g. $Z = \frac{(X - \mu)}{\sigma}$
• recall conditions under which the normal distribution can be used as an approximation to the binomial distribution, and use this approximation, with a continuity correction, in solving problems.	n sufficiently large to ensure that both $np > 5$ and $nq > 5$.

PROBABILITY & STATISTICS

Summary statistics

For ungrouped data:

$$\bar{x} = \frac{\Sigma x}{n}, \qquad \text{standard deviation } = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$$

For grouped data:

$$\bar{x} = \frac{\Sigma xf}{\Sigma f}, \qquad \text{standard deviation } = \sqrt{\frac{\Sigma(x-\bar{x})^2 f}{\Sigma f}} = \sqrt{\frac{\Sigma x^2 f}{\Sigma f} - \bar{x}^2}$$

Discrete random variables

$$\mathrm{E}(X) = \Sigma xp, \qquad \mathrm{Var}(X) = \Sigma x^2 p - \{\mathrm{E}(X)\}^2$$

For the binomial distribution $\mathrm{B}(n, p)$:

$$p_r = \binom{n}{r} p^r (1-p)^{n-r}, \qquad \mu = np, \qquad \sigma^2 = np(1-p)$$

For the geometric distribution Geo(p):

$$p_r = p(1-p)^{r-1}, \qquad \mu = \frac{1}{p}$$

For the Poisson distribution $\mathrm{Po}(\lambda)$

$$p_r = \mathrm{e}^{-\lambda}\frac{\lambda^r}{r!}, \qquad \mu = \lambda, \qquad \sigma^2 = \lambda$$

Continuous random variables

$$\mathrm{E}(X) = \int x\,\mathrm{f}(x)\,\mathrm{d}x, \qquad \mathrm{Var}(X) = \int x^2\,\mathrm{f}(x)\,\mathrm{d}x - \{\mathrm{E}(X)\}^2$$

Sampling and testing

Unbiased estimators:

$$\bar{x} = \frac{\Sigma x}{n}, \qquad s^2 = \frac{\Sigma(x-\bar{x})^2}{n-1} = \frac{1}{n-1}\left(\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right)$$

Central Limit Theorem:

$$\bar{X} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right)$$

Approximate distribution of sample proportion:

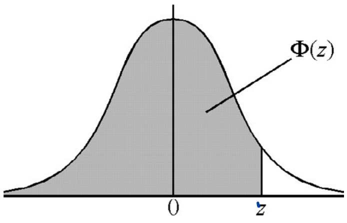
$$\mathrm{N}\left(p, \frac{p(1-p)}{n}\right)$$

THE NORMAL DISTRIBUTION FUNCTION

If Z has a normal distribution with mean 0 and variance 1, then, for each value of z, the table gives the value of $\Phi(z)$, where

$$\Phi(z) = \mathrm{P}(Z \leqslant z).$$

For negative values of z, use $\Phi(-z) = 1 - \Phi(z)$.



z	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
											ADD								
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	2	5	7	9	12	14	16	19	21
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	2	4	6	8	10	12	14	16	18
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	2	4	6	7	9	11	13	15	17
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	2	3	5	6	8	10	11	13	14
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1	3	4	6	7	8	10	11	13
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	1	2	4	5	6	7	8	10	11
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	1	2	3	4	5	6	7	8	9
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	1	2	3	4	4	5	6	7	8
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	1	1	2	3	4	4	5	6	6
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	1	1	2	2	3	4	4	5	5
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0	1	1	2	2	3	3	4	4
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0	1	1	2	2	2	3	3	4
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0	1	1	1	2	2	2	3	3
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0	1	1	1	1	2	2	2	2
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0	0	1	1	1	1	1	2	2
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0	0	0	1	1	1	1	1	1
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0	0	0	0	1	1	1	1	1
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0	0	0	0	0	1	1	1	1
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0	0	0	0	0	0	0	1	1
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0	0	0	0	0	0	0	0	0

0.6311
19
0.6330

Critical values for the normal distribution

If Z has a normal distribution with mean 0 and variance 1, then, for each value of p, the table gives the value of z such that

$$\mathrm{P}(Z \leqslant z) = p.$$

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291